

**Adaptive control of the
singularly perturbed chaotic systems
based on the scale time estimation
by keeping chaotic property**

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Abstract

In this paper, a new approach to the problem of stabilizing a chaotic system is presented. In this regard, stabilization is done by sustaining chaotic properties of the system. Sustaining the chaotic properties has been mentioned to be of importance in some areas such as biological systems.

The problem of stabilizing a chaotic singularly perturbed system will be addressed and a solution will be proposed based on the OGY (Ott, Grebogi and Yorke) methodology. For the OGY control, Poincare section of the system is defined on its slow manifold. The multi-time scale property of the singularly perturbed system is exploited to control the Poincare map with the slow scale time. Slow scale time is adaptively estimated using a parameter estimation technique. Control with slow time scale circumvents the need to observe the states. With this strategy, the system remains chaotic and chaos identification is possible with online calculation of Lyapunov exponents.

Using this strategy on ecological system improves their control in three aspects. First that for ecological systems sustaining the dynamical property is important to survival of them. Second it removes the necessity of insertion of control action in each sample time. And third it introduces the sufficient time for census.

Keywords: OGY, Lyapunov exponent, slow manifold, adaptive, singular perturbation, scale time

1. Introduction

Nonlinear singularly perturbed models are known by dependence of the system properties on the perturbation parameter [5]. Multi time scale characteristic is an important property of this class of nonlinear systems. For this class of systems a two-stage procedure for design composite controller is presented in [9]. On the other hand, chaotic behavior is an important characteristic of a class of nonlinear systems. Many researchers have shown interest in the analysis and control of the chaotic systems. Among the proposed approaches is the control of the Poincare Map (the OGY-Method) [8].

In this paper, the OGY method is applied to the singularly perturbed chaotic systems. The proposed control strategy exploits the chaotic property of the system and a discrete system model on the Poincare map is defined. This Poincare map lies on the slow manifold of the system. It is shown that by using the two time scale property of the system, an OGY control with slow time scale on the slow manifold of system, could be defined.

This strategy of control results in keeping chaotic property of the system and then online identification of chaos with calculation of Lyapunov exponents is possible. An adaptive parameter estimation technique is used to estimate perturbation parameter and the slow time scale of the singularly perturbed system. Population models are examples of systems where sustaining the dynamical property of controlled systems is important for survival of them. Chaotic model of food chains were initially found in [2,4]. Recently, chaotic impulsive differential equations are used in biological control [6,11-13]. Multi time scale approach was first used in [7] for food chain models. Method is implemented on a prey-predator type of population model.

The paper is organized as follows. In section 2 the slow-fast manifold separation based on the slow and fast states for singularly perturbed systems is introduced.

In section 3 an adaptive estimation technique for the estimation of perturbation parameter is proposed.

In section 4 chaotic property of the system is exploited and the OGY control is implemented for the stabilizing problem. Then singularly perturbed property is exploited and slow manifold of the system is selected as the Poincare section. Then a new control based on the slow scale time estimation is introduced.

Section 5 presents the results of employing the proposed method on the ecological prey-predator system.

2. Problem Formulations

In this paper, chaotic singularly perturbed systems of the following form are considered,

$$\begin{aligned} \varepsilon \dot{x} &= f(x, y) \\ \dot{y} &= g(x, y) \end{aligned} \quad (1)$$

Where, $x \in R$, $y \in R^{n-1}$ and ε is a small parameter. $f : R^n \rightarrow R$, $g : R^n \rightarrow R^{n-1}$ are both smooth functions and the system is chaotic. The slow manifold of (1) is defined with

$$\begin{aligned} 0 &= f(x, y) \\ \dot{y} &= g(x, y) \end{aligned} \quad (2)$$

This S manifold $S : \{f = 0\}$ is smooth and results in separation of time scales as x the fast, and y as the slow variable. It is easily seen that,

$$\frac{\dot{y}}{\dot{x}} = \frac{g(x, y)}{\varepsilon f(x, y)} \xrightarrow{\varepsilon \ll 1} \frac{\dot{y}}{\dot{x}} \propto \frac{1}{\varepsilon} \quad (3)$$

By taking

$$\tau = \frac{t}{\varepsilon} \quad (4)$$

the second scale time of system is $T = \frac{1}{\varepsilon}$.

3. Adaptive Estimation of ε

In [10] an estimation method for constant terms using the least-squares approach is proposed. Here the method is used here for ε estimation. The estimated ε is found to minimize the total prediction error as

$$J = \int_0^t e^2(r) dr$$

Where the prediction error $e(t)$ is defined as

$$e(t) = \hat{\varepsilon} g(x, y) \dot{x} - f(x, y) \dot{y}$$

This total error minimization can average out the effects of measurement noise. The resulting estimation is [4]

$$\hat{\varepsilon} = \frac{\int_0^t \dot{x} g(x, y) \dot{y} f(x, y) dr}{\int_0^t \dot{x}^2 g(x, y)^2 dr} \quad (5)$$

To reduce the size of manipulations we defined w window, then (5) changes to (6)

$$\hat{\varepsilon} = \frac{\int_{t-w}^t \dot{x} g(x, y) \dot{y} f(x, y) dr}{\int_{t-w}^t \dot{x}^2 g(x, y)^2 dr} \quad (6)$$

4. OGY Control Based On Second Time Scale Estimation

In this part a new control strategy is proposed such that controlled system remains chaotic. This strategy exploits OGY method to design control and then uses two time scale property of the system to improve the designed control such that system remains chaotic.

4.1 Fast Direction Properties

Consider the chaotic singularly perturbed system (1). As ε is a small parameter, an approximation of the fastest eigenvalue of Jacobian matrix (7) is $\frac{1}{\varepsilon} \times \frac{\partial g}{\partial z}$. Since the chaotic systems are dissipative and the absolute value of the sum of negative exponents is bigger than the sum of the positive Lyapunov exponents, this big value is almost negative. It means that calculation of Lyapunov exponents in fast direction is not necessary in chaos identification. And for system with one perturbation term the fast direction is a stable direction.

$$J = \begin{bmatrix} \frac{1}{\varepsilon} \times \frac{\partial f}{\partial x} & \dots & \dots & \frac{1}{\varepsilon} \times \frac{\partial f}{\partial y_{n-1}} \\ \frac{\partial g_1}{\partial x} & \frac{\partial g_1}{\partial y_1} & \dots & \frac{\partial g_1}{\partial y_{n-1}} \\ \vdots & \dots & \dots & \vdots \\ \frac{\partial g_{n-1}}{\partial x} & \frac{\partial g_{n-1}}{\partial y_1} & \dots & \frac{\partial g_{n-1}}{\partial y_{n-1}} \end{bmatrix} \quad (7)$$

4.2 OGY Control On The Slow Manifold

In the OGY control design, a manifold is defined such that the discrete model of the system will be obtained by the intersections of this manifold with system trajectories. Then, the control of this discrete model on this manifold will result in the control of the system. It is obvious that the manifold approach will result in a more accurate control of system if it contains all unstable modes of the system. Stable modes lead the system dynamics toward a desired point. One of the modes should be eliminated to have a manifold with unit co-dimension. Eliminating the fastest stable mode and letting it to be free leads to a more accurate control (compared to the elimination of other modes).

Considering Jacobian matrix (7) Where x_{eq} is the value on the fixed point of the system while the fixed point is calculated as:

$$\begin{aligned} f(x_{eq}, y_{eq}) &= 0 \\ g(x_{eq}, y_{eq}) &= 0 \end{aligned} \quad (8)$$

Then, the discrete model will be:

$$y_{k+1} = p(y_k, u_k), x = x_{eq} \quad (9)$$

The OGY control, u_k proposed with the following strategy on slow manifold will be:

$$u_k = \begin{cases} K(y_k) & \text{if } |y_k - y_{eq}| \leq \Delta \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

Where Δ is the dead zone in traditional OGY method and $K(y_k)$ is a control for slow states of discrete model (9) designed with a suitable method for example, proportional feedback.

4.3 New OGY Control Based On Slow Time Scale Estimation

In the OGY method, control of the Poincare map is equivalent to the control of the chaotic system (1). According to two time scale property of the system, to control this Poincare section on the slow manifold, it is sufficient to control it with the slow scale time, because the states on the slow manifold have slower motions than total dynamical system. Hence, control is designed by following strategy: Control starts with the OGY control and as soon as the first section with the Poincare map is detected, system could be controlled with the slow scale time.

Suppose that T is the estimation of slow scale time of the system. And k_0 is the time of first section or first pulse, the system can be controlled by inserting control action (10) only in the following instants;

$$k = k_0 + nT, n = 0, 1, 2, \dots$$

Where, according to (4) T is

$$T = \left[\frac{1}{\varepsilon} \right] \quad (11)$$

and $[\dots]$ is a bracket symbol.

an approximation of the fastest mode will be λ_1 . Then the accurate manifold is defined by the equation;

$\Sigma = \{ x \mid \lambda_1(x) = 0 \}$ which is the slow manifold of the system. In other expression with this strategy, the Poincare section in this problem lies on the slow manifold (2). For stabilizing problem in fixed point Poincare section becomes:

$$(12)$$

The main idea of this new control method is keeping the system on its chaotic state without resisting to be settled down in the desired rejoin.

The control strategy can be summarized as; when the chaotic system states enter the dead zone, by insertion of the control pulse, the states settle more in the neighborhood of the slow manifold. Afterward system works in open loop and remains by its dynamic in the slow manifold. This slow manifold contains all unstable modes that are also all slow. If unstable modes try to abduct trajectory from the desired point, it needs a time. Since smaller ε result in bigger fast stable modes then the time that system remains on slow manifold increases too, an approximation of this time is slow time scale of the system.

During this time after the application of the control pulse, states of the system remain in the neighborhood of the desired trajectory. Then, after this time before the exit from the desired region, the loop is closed again and insertion of an enough effective control pulse returns the trajectories closer in the slow manifold. Then system becomes open loop again and so on.

Result 1: control of Poincare map and control of system (1) are equivalent. With T period the system is controlled. Then all needed information to control the Poincare map exist at k_0 . Then T is the sufficient census time (sufficient period of observation) for system (1).

Result 2: By this method between the pulses system is open loop. Then system remains chaotic and online calculation of the Lyapunov exponents result in positive maximum lyapaunov exponent. By defining λ_1 as

$$(13)$$

When $\lambda_1 < 0$ the system identified as chaotic and control rule (10) could be inserted adaptively.

