Wave Fractal Dimension as a Tool in Detecting Cracks in Beam Structures

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Abstract. A chaotic signal is used to excite a cracked beam and wave fractal dimension of the resulting time series and power spectrum are analyzed to detect and characterize the crack. For a single degree of freedom (SDOF) approximation of the cracked beam, the wave fractal dimension analysis reveals its ability to consistently and accurately predict crack severity. For a finite element simulation of the cracked cantilever beam, an analysis of spatio-temporal response using wave fractal dimension in frequency domain reveals distinctive variation vis-à-vis crack location and severity. Simulation results are experimentally validated.
Keywords: Chaotic excitation, Chen's oscillator, Wave fractal dimension.

1 Introduction

Vibration-based methods for crack detection in beam type structures continue to attract intense attention from researchers. To quantify the crack depth and to detect crack location, vibration-based crack detection methods employ a variety of characterizing parameters, such as natural frequency [8], mode shape [13], mechanical impedance [2], statistical parameters [16], etc. In recent research, wave fractal dimension, originally introduced by Katz [9] to characterize biological signals, has been used to detect the severity and location of crack in beam [4] and plate structures [5].

Since the past decade, progress in chaos theory has led several researchers to consider the use of chaotic excitation in vibration-based crack detection [11,12]. A majority of these efforts necessitate the reconstruction of a chaotic attractor from the time series data corresponding to the vibration response of the structure [11,12]. Unfortunately, the reconstruction of a chaotic attractor is often tedious and may not always yield satisfactory results for crack detection even in the SDOF approximation case. To detect and characterize cracks, the current chaos-based crack detection methods use a variety of chaos and statistics-based parameters, such as correlation dimension [12], Hausdorff distance [12], average local attractor variance ratio [11], etc. In this paper, we study the use of wave fractal dimension as a characterizing parameters to predict the severity and location of a crack in a beam that is made to vibrate using a chaotic input.

2 Beam Excitation Input

In [3] we considered three methods to excite the cracked beam: a non-zero initial condition, a harmonic input, and the chaotic solution of autonomous dissipative flow type Chen's attractor [14]. Due to space constraints, here we report on the results corresponding to the use of chaotic signal as an input excitation force to vibrate a cracked beam. The Chen's system in state space form is expressed as

$$\dot{y_1} = a_1(y_2 - y_1), \quad \dot{y_2} = (a_3 - a_1)y_1 - y_1y_3 + a_3y_2, \quad \dot{y_3} = y_1y_2 - a_2y_3, \tag{1}$$

where a_1, a_2, and a_3 are constant parameters. Setting the parameters to $a_1 = 35$, $a_2 = 3$, and $a_3 = 28$, with initial values of $y_1(0) = -10$, $y_2(0) = 0$, and $y_3(0) = 37$, equation (1) exhibits a chaotic behavior [3,14] and the solution y_1 is expected to be non-periodic. See [3] for our reasons to restrict consideration to Chen's system.

3 Wave Fractal Dimension

Waveforms are common patterns that arise frequently in scientific and engineering phenomena. The concept of wave fractal dimension [9] is used to differentiate one waveform from another. For waveforms, produced using a collection of ordered point pairs (x_i, y_i), $i = 1, \ldots, n$, the total length, L, is simply the sum of the distances between successive points, i.e., $L = \sum_{i=1}^{n-1} \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}$. Moreover, the diameter d of a waveform is considered to be the farthest distance between the starting point (corresponding to $n = 1$) and some other point (corresponding to $n = i$, $i = 2, \ldots, n$), of the waveform, i.e., $d = \max_{i=2,\ldots,n} \sqrt{(x_i - x_1)^2 + (y_i - y_1)^2}$. Next, by expressing the length of a waveform L and its diameter d in a standard unit, which is taken to be the average step α of the waveform, the wave fractal dimension can be expressed as [9]

$$D = \log(L/\alpha)/\log(d/\alpha) = \frac{\log(n)}{\log(n) + \log(d/L)}, \tag{2}$$

where $n = L/\alpha$, denotes the number of steps in the waveform. We use (2) to estimate the wave fractal dimension.

4 Modeling of a Cracked Beam as a SDOF System with Force Input

Following [1,12], a cracked beam is modeled as a SDOF switched system which emulates the opening and closing of the surface crack by switching the effective stiffness $k_s = k - \Delta k$, where k is the stiffness of the beam without crack, k_s is stiffness during stretching and Δk is stiffness difference. For a SDOF model with a relatively small crack, the ratio of Δk to k is equal to the ratio of the crack depth a to the thickness h of the beam [1,12]. Next, we consider that the y_1 solution of (1) is applied as a force to the mass of the SDOF system. The equations of motion for this piecewise continuous SDOF system are

$$\begin{aligned} M\ddot{x} + c\dot{x} + kx &= F(t), \qquad \text{for } x \geq 0, \\ M\ddot{x} + c\dot{x} + k_s x &= F(t), \qquad \text{for } x < 0, \end{aligned} \tag{3}$$

where M is the mass of the cantilever beam, c is the damping coefficient, and x is the displacement of the beam. The physical parameters of the problem data used in our simulations are as follows: mass $m = 0.18$ kg, nominal stiffness $k = 295$ N/m, and damping $c = 0.03$ Ns/m.

5 SDOF Results

We now consider the application of the chaotic forcing input of (1) to vibrate the SDOF model for various values of crack depths. The resulting time series plots (see Figure 3.11 of [3]) are used to compute the corresponding wave fractal dimension. Figure 1(a) plots normalized crack depth versus the wave fractal dimension, showing that the wave fractal dimension monotonically increases with increasing crack depth. Since wave fractal dimension is a characteristic of the waveform only, we consider the wave fractal dimension analysis of the time series of [3] in frequency domain. Using the Fast Fourier Transform (FFT) [7] technique, we convert the time domain data of [3] to frequency domain (see Figure 3.13 of [3]), producing the power spectrum of the response of the SDOF cracked beam. The power spectrum reveals that the portion of FFT in the vicinity of beam's natural frequency ω_n experiences significant changes. Thus, we concentrate in the neighborhood of ω_n as our window for computing the wave fractal dimension. Using this technique, in Figure 1(b), we plot normalized crack depth versus the wave fractal dimension for the windowed waveforms of Figure 3.13 of [3]. The wave fractal dimension is seen to monotonically increase with increasing crack depth and this curve exhibits a significant rate of change. Thus, in the following analysis, we use the wave fractal dimension of power spectrum as a natural choice for crack detection and crack characterization.

6 Continuous Model

We now extend the results of section 5 to the continuous model case. To do so, as in [13,15], we consider a continuous model of the dynamical behavior of the beam with a surface crack in two parts. Specifically, when the beam

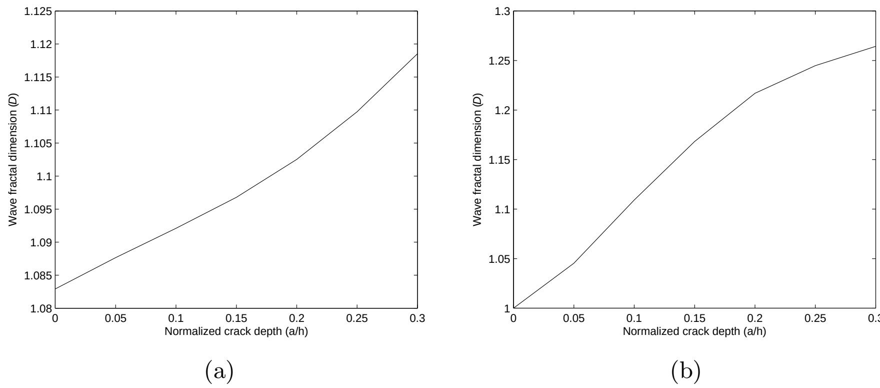


Fig. 1. (a) Time domain and (b) Frequency domain change of the wave fractal dimension with normalized crack depth for chaotic forcing input

moves away from the neutral position so that the crack remains closed, then the beam behaves as a typical continuous beam [3,13,15]. However, when the beam moves in the other direction from the neutral position, causing the crack to open, the resulting dynamics require the modeling of crack with a rotational spring whose stiffness is related to the crack depth [2,3,13,15].

Next, we used the ANSYS software [10] to simulate the dynamics of a cracked beam under external excitation. We modeled the beam as a 2-D elastic object using a *beam3* element [10] which has tension, compression, and bending capabilities. The crack is simulated by inserting a torsional spring at the location of the crack and using the mathematical model described in [2,3,13,15]. The torsional spring is modeled using a *combin14* element [10] which is a spring-damper element used in 1-D, 2-D, and 3-D applications. In our FE model, we used the *combin14* element as a pure spring with 1-D (i.e., torsional) stiffness since the model of [2,3,13,15] does not consider damping. The physical characteristics of the beam used in our FE model are as follows: material–Plexiglass, length–500 mm, width–50 mm, thickness–6 mm, modulus of elasticity–3300 MPa, density–1190 kg/m^3, and Poisson's ratio–0.35. This FE model was validated [3] by comparing the natural frequencies resulting from the FE simulations versus the natural frequencies computed in Matlab for the dynamic model of [3,13,15].

Next, we apply force input to the FE model using the time series y_1 of (1). In particular, using MATLAB, we simulate (1) and save $15,000$ time steps of y_1 time series, which is applied as force input at 40 mm from the fixed end in ANSYS. The FE simulation is used to produce and record spatio-temporal responses for each node (corresponding to discretized locations along the beam span). The resulting data is imported in MATLAB for a detailed wave fractal dimension analysis, whose results are grouped in two parts as explained below.

We first analyze the beam tip displacement power spectrum data to detect the presence of any cracks along the beam span. Figure 2 provides plots of the normalized crack depth a/h versus wave fractal dimension for a crack located at $L_1 = 0.2L$ and, alternatively, at $L_1 = 0.4L$. We term these curves as *uniform crack location curves*. We observe that a beam without a crack yields a wave fractal dimension of 1.1205, and wave fractal dimension above this nominal value indicates presence of a crack in the beam. However, it is not possible to determine either the crack depth or crack location using only the beam tip response analysis.

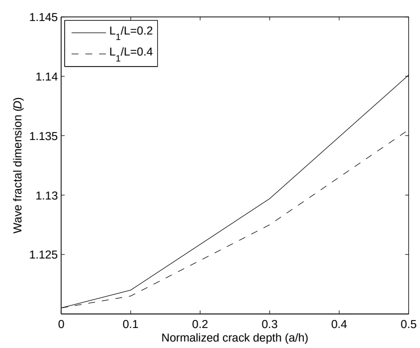
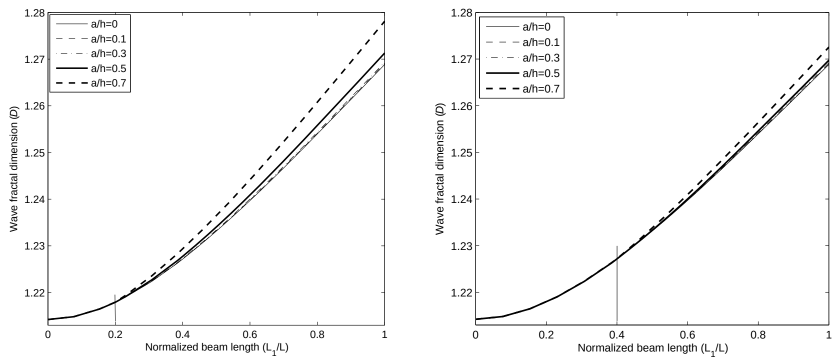


Fig. 2. Wave fractal dimension versus normalized crack depth–uniform crack location curves for $L_1 = 0.2L$ and $L_1 = 0.4L$

Next, to predict the severity and approximate location of the crack on the beam surface, we record the time series data of the beam response along its span for chaotic forcing input. Using the FFT, the time series data is converted to frequency domain. The resulting power spectrum plot is analyzed to identify a suitable window for computing the wave fractal dimension. Throughout this analysis, the frequency window used for computing the wave fractal dimension is kept fixed for all crack depths considered. Figure 3(a) plots wave fractal dimension against normalized beam length for cracks of various severity located at $L_1 = 0.2L$. These *uniform crack depth curves* yield the same wave fractal dimension till the crack location and their slopes change abruptly at the location of crack. In fact, past the crack location, the uniform crack depth curves exhibits a larger slope for a larger crack depth. Figure 3(b) shows similar behavior for crack location, $L_1 = 0.4L$. The abrupt split in uniform crack depth curves at crack location and their increasing slope with increasing crack depth can be used to establish both the severity and location of crack.



(a) (b)

Fig. 3. Wave fractal dimension versus normalized beam length–uniform crack depth curves for (a) $L_1 = 0.2L$ and (b) $L_1 = 0.4L$

7 Experimental Verification

A schematic of the experimental setup used is given in Figure 4. An aluminum base holds the shaker (Brüel & Kjær Type 4810). To produce a base excitation, a test specimen is clamped on shaker. An accelerometer (Omega ACC 103) is mounted at the tip of the specimen using mounting bee wax. Our software environment consists of Matlab, Simulink, and Real Time Workshop in which the Chen's chaotic oscillator is propagated to obtain the time series corresponding to the y_1 signals of (1). Next, an analog output block in the Simulink program outputs the y_1 signal to a digital to analog converter of Quanser's Q4 data acquisition and control board which in turn is fed to a 12 volt amplifier (Kenwood KAC-8202) to drive the shaker. The accelerometer output is processed by an amplifier (Omega ACC PSI) and interfaced to an analog to digital converter of the Q4 board for feedback to the Simulink program. Properties of the specimen used in our experiments are same as in Section 6. To emulate a fine hair crack, we used a 0.1 mm saw to introduce cracks of several different desired depths. For specimen of different crack depth, all located at $L_1 = 0.2L = 100$ mm from fixed end, the accelerometer measurement is recorded and used to produce the output response time series, which is used to perform our analysis. A total of six specimens were prepared with crack depth varying from 0% to 50% of the thickness. In all the specimen, saw crack was introduced on the top surface to match with the simulation condition.

The time series data obtained from the accelerometer suffered from general sensor errors (dc offset and ramp bias), causing the raw time series data to be unusable for further analysis. We used the Wavelet transformation toolbox [6] of MATLAB, to filter the raw time series data and remove the errors [3]. The corrected time series data [3] is converted to frequency domain and used to compute wave fractal dimension. Figures 5 provides the variation in wave fractal dimension versus the crack depth for the corrected power spectrum data. Note that the wave fractal dimension shows an increasing trend with increasing crack depth validating the predictions of our numerical study in Section 5 for SDOF case and in Section 6 for the continuous beam case when only tip displacement measurement is used. Although the plots obtained from the experimental data are not as smooth as the ones resulting from numerical simulation, this may be the result of inaccuracies resulting from sample preparation or a variety of experimental errors [3].

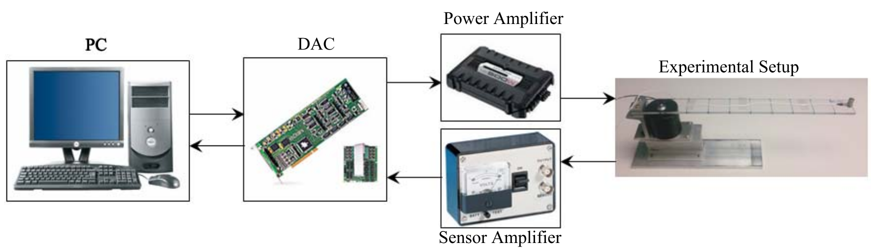


Fig. 4. Experimental setup

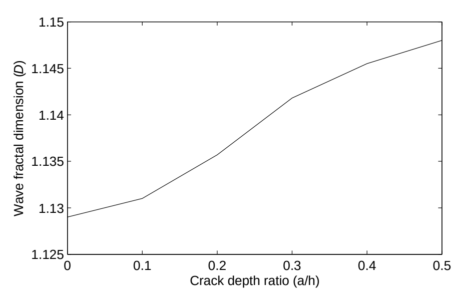


Fig. 5. Frequency domain change of wave fractal dimension with normalized crack depth at $L_1 = 0.2L$

8 Conclusion

In this paper, to detect and characterize a crack in a beam, we considered a SDOF and a FE model of the beam excited by a chaotic force input. We showed that for the SDOF model, crack severity can be easily and consistently predicted by using wave fractal dimension of power spectrum of time series data. Moreover, for the FE model, we showed that wave fractal dimension exhibits a trend that can be used to predict crack location and crack depth. Finally, the simulation results were validated experimentally.

Acknowledgments

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