Pattern Formation of the Stationary Cahn-Hilliard Model

Hansjörg Kielhöfer

Universität Augsburg, Germany

We investigate critical points of the free energy of the Cahn-Hilliard model of a binary alloy under the constraint of a constant mass. The domain is the unit square. Minimizers of the energy without interfacial energy term are given by a decomposition of the two components of the alloy, but the interfaces between the components are arbitrary. Specific patterns are only formed if an interfacial energy term is present. We select such patterns of minimizers by an approximation of sequences of conditionally critical points of the free energy when the interfacial energy term tends to zero. This is what we call Pattern Formation of the Stationary Cahn-Hilliard Model. Mathematically it is a singular limit process.

We obtain the conditionally critical points by a global bifurcation analysis of the Euler-Lagrange equation for the free energy where the mass is the bifurcation parameter. The eigenvalues of the linearization at the trivial solution, which is given by a homogeneous mixture, define the bifurcation points. The corresponding eigenfunctions have characteristic symmetries over the unit square, and the elliptic maximum principle implies that these symmetries, in particular the location of the maxima and minima, are fixed for all solutions of the nonlinear equation on global bifurcating branches. These properties allow a uniform a priori estimate of the branches and also the existence of a singular limit as the interfacial energy term tends to zero. This limit is a nontrivial conditionally critical point and by its symmetries it fulfills the Weierstrass-Erdmann corner conditions known from one-dimensional calculus of variations. Therefore the limit is a minimizer of the free energy without interfacial energy term whose pattern is formed by the singular limit process. This process can be done for all bifurcating branches and for all masses in the so-called spinodal region.

4th Chaotic Modeling and Simulation International Conference, CHAOS2011
31 May – 3 June 2011, Agios Nikolaos, Crete, Greece