# **CHAOS 2020**

# **Book of Abstracts**

# 13<sup>th</sup> Chaotic Modeling and Simulation International Conference

Editor

Christos H. Skiadas



9-12 June, 2020

## **Imprint**

Book of Abstracts of the 13th Chaotic Modeling and Simulation International Conference (9-12 June, 2020)

Published by: ISAST: International Society for the Advancement of

Science and Technology. Editor: Christos H Skiadas

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# **Preface**

# 13<sup>th</sup> Chaotic Modeling and Simulation International Conference

9 – 12 June 2020

It is our pleasure to welcome the guests, participants and contributors to the 13<sup>th</sup> International Conference (CHAOS2020) on Chaotic Modeling, Simulation and Applications. We support the study of nonlinear systems and dynamics in an interdisciplinary research field and very interesting applications will be presented. We intend to provide a widely selected forum to exchange ideas, methods, and techniques in the field of Nonlinear Dynamics, Chaos, Fractals and their applications in General Science and in Engineering Sciences.

The principal aim of CHAOS2020 International Conference is to expand the development of the theories of the applied nonlinear field, the methods and the empirical data and computer techniques, and the best theoretical achievements of chaotic theory as well.

Chaotic Modeling and Simulation Conferences continue to grow considerably from year to year thus making a well established platform to present and disseminate new scientific findings and interesting applications.

We thank all the contributors to the success of this conference and especially the authors of this *Book of Abstracts*. Special thanks to the Plenary, Keynote and Invited Presentations, the Scientific Committee, the ISAST Committee, Yiannis Dimotikalis and Aris Meletiou and the web supporting team, the Conference Secretary Eleni Molfesi and all the members of the Secretariat.

May 2020

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# Plenary – Keynote – Invited Speakers

### Nail Akhmediev

Department of Theoretical Physics, Australian National University *Recent advances in rogue wave theory* 

### Elena Babatsouli

Communicative Disorders Department, University of Louisiana at Lafayette, USA

Order in disordered speech data

### Mark Edelman

Department of Physics, Stern College at Yeshiva University, New York, NY, USA

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Evolution of Systems with Power-Law Memory: Do We Have to Die?

### Jean-Marc GINOUX

Laboratoire d'Informatique et des Systèmes, LIS. Université de Toulon, France Archives Henri Poincaré, UMR CNRS 7117

Albert Einstein and the doubling of the deflection of light

### Nikolav V. Kuznetsov

Department of Applied Cybernetics, Saint-Petersburg State University, Institute for Problems in Mechanical Engineering of the Russian Academy of Sciences

Theory of hidden oscillations (with a tribute to Gennady A. Leonov)

### Wieslaw M. Macek

Faculty of Mathematics and Natural Sciences Cardinal Stefan Wyszynski University, Warsaw, Poland and Space Research Centre, Polish Academy of Sciences, Warsaw, Poland Reconnection and Turbulence in Space Plasmas on Kinetic Scale

## Riccardo Meucci

Istituto Nazionale di Ottica – CNR, Largo E. Fermi 6, 50125 Firenze, Italy *Recent Advances in Controlling Chaos* 

### Leszek Sirko

Institute of Physics, Polish Academy of Sciences, Warsaw, Poland What can we learn from the spectra of quantum graphs and microwave networks?

## **Banlue Srisuchinwong**

Sirindhorn International Institute of Technology, Thammasat University, Thailand

A Simplest 1-BJT-Based Chaotic Hyperjerk Circuit: Its Minimized Damping for Maximized Attractor Dimension, and Hidden Attractors

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# **CHAOS 2020**

# **Book of Abstracts**

# 13<sup>th</sup> Chaotic Modeling and Simulation International Conference

# **Plenary and Keynote Talks**

## Recent advances in rogue wave theory

### **Nail Akhmediev**

Department of Theoretical Physics, Australian National University

## Order in disordered speech data

### Elena Babatsouli

Department of Communicative Disorders University of Louisiana at Lafayette

This presentation is concerned with the developmental aspects of speech acquisition by children and, specifically, the acquisition of consonants, the main source of speech errors. An important criterion that distinguishes typically developing children from children with speech sound disorders is the level of consonant acquisition in relation to age. A particular difficulty children have is to produce consonants adjacent to other consonants, forming consonants clusters. Generally, consonant clusters are acquired later than the individual cluster members. The widely used measure in the literature of consonants correct does not account for the different stages of cluster productions, resulting in misevaluation of a child's acquisition level. In the present work, my recently proposed measure for cluster

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accuracy is applied to children with typical development and also to children with speech sound disorders. Subsequently this measure is compared to the measure of consonants correct. Special attention is paid to the developmental stages of a bilingual child whose two measures undergo an 8-month stage where succeeding measure values form a Poincare-like map with a strange attractor. While the measures are distributed disorderly per consonant cluster within and across children, they are orderly correlated to each other. These results demonstrate the importance of using a special measure to quantitatively evaluate consonant clusters.

# Evolution of Systems with Power-Law Memory: Do We Have to Die?

### Mark Edelman

Stern College at Yeshiva University, New York, NY, USA; Courant Institute of Mathematical Sciences, New York, NY, USA; BCC, CUNY, New York, NY, USA

Various features of the development of individual living species, including individual humans, are programmed. Is death also programmed, and if ves, how is itimplemented and what can be the underlying mechanism providing the inevitability of death? The hypothesis presented in this presentation is based on the similarity of the human evolution to the evolution of simple discrete nonlinear fractional (with power-law memory) systems. Caputo fractional/fractional difference logistic map is a simple discrete system with power-/asymptotically power-law memory and quadratic nonlinearity. In the area of parameters where the fixed point is unstable, its evolution starts as the evolution of a system with a stable fixed point but then this fixed point becomes unstable, suddenly breaks, and turns into a period two point. Considered under various types of random perturbations, the time spans of the evolution as a fixed point before the break (lifespans) obey the Gompertz-Makeham law, which is the observed distribution of the lifespans of living species, including humans. The underlying reason for modeling the evolution of humans by fractional systems are the observed power law in human memory and the viscoelastic nature of organ tissues of living species. Models with powerlaw memory may explain the observed decrease at very large ages of the rate of increase of the force of mortality and they imply limited lifespans. Keywords: Chaotic modeling, lifespan distribution, Gompertz-Makeham law, power-law memory, fractional dynamics, bifurcations.

# Albert Einstein and the doubling of the deflection of light

### Jean-Marc Ginoux<sup>1</sup>

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One of the three consequences of Einstein's theory of general relativity was the curvature of light passing near a massive body. In 1911, he published a first value of the angle of deflection of light, then a second value in 1915, equal twice the first. In the early 1920s, when he received the Nobel Prize in Physics, a violent controversy broke out over this result. It was then disclosed that the first value he had obtained in 1911 had been calculated more than a century before by a German astronomer named Johann von Soldner. The aim of this article is therefore to compare the methods used by Soldner and then by Einstein leading to this first value and to explain the importance of the doubling of this value in the framework of Einstein's theory of gravitation.

# Theory of hidden oscillations (Dedicated to Gennady Alekseevich Leonov (1947-2018))

### Nikolay V. Kuznetsov

<sup>1</sup>Saint-Petersburg State University, Russia <sup>2</sup>University of Jyvaskyla, Finland <sup>3</sup>Institute for Problems in Mechanical Engineering RAS, Russia

# Reconnection and Turbulence in Space Plasmas on Kinetic Scales

### Wieslaw M. Macek<sup>1,2</sup>

<sup>1</sup>Faculty of Mathematics and Natural Sciences Cardinal Wyszyński University, Warsaw, Poland, <sup>2</sup>Space Research Centre, Polish Academy of Sciences, Warsaw, Poland

Turbulent magnetic fields play an important role in plasmas leading to magnetic reconnection and the redistribution of kinetic and magnetic energy in space environments and laboratory plasmas. Reconnection occurs when the electrons cannot supply the current needed to support antiparallel magnetic fields. This is a complex phenomenon that still remains a challenge for contemporary physics. We have already considered magnetic turbulence using observations from Magnetospheric Multiscale (MMS) mission on kinetic (ions and electron) scales, which are far shorter than the scales characteristic for description of plasma by magnetohydrodynamic (MHD) theory. In particular, we have shown that a clear break of the magnetic spectral exponent to about -11/2 at frequencies 20 - 25 Hz agrees with the predictions of kinetic theory (-16/3) Ref. [1]. It is worth noting that the unprecedented very high (millisecond) resolution of the magnetic field instrument enabled us to grasp the mechanism of reconnection in the magnetotail on ion and electron scale lengths [2]. As expected from numerical simulations, we have verified that when the field lines and plasma become decoupled a large reconnecting electric field related to the Hall current (1–10 mV/m) is responsible for fast reconnection in the ion diffusion region both at the magnetopause and in the magnetotail regions. Although inertial accelerating forces remain moderate (1-2 mV/m), the electric fields resulting from the divergence of the full electron pressure tensor provide the main contribution to the generalized Ohm's law at the neutral sheet (of the order of 100 mV/m), see Ref. [3]. In our view, this illustrates that when ions decouple electron physics dominates. The results obtained on kinetic scales may be useful for better understanding the physical mechanisms governing reconnection processes in various magnetized plasmas in the Universe.

**Keywords:** Reconnection, Turbulence, Space plasmas, Kinetic theory.

## **Recent Advances in Controlling Chaos**

### Riccardo Meucci

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The seminal paper on chaos control by Ott, Grebogi and Yorke in 1990 opened a whole new area of research introducing the concept that chaos can be controlled and manipulated by applying tiny perturbations [1]. Few years later, in 1994 we proposed an open loop method for chaos control in a modulated laser. The key feature of our method is the phase difference between the fundamental frequency inducing chaos and the perturbing frequency inducing stabilization of different periodic orbits contained in a chaotic attractor [2]. Such a control method was later named "Phase control of Chaos" and it has been successfully tested in different non- autonomous chaotic systems including the FitzHugh-Nagumo neuron model [3]. Recently such a control strategy was applied to the driven Duffing's oscillator emphasizing on the differences as the phased perturbation is applied to the linear term, the cubic term and the driving term of the oscillator equation and at the same time confirming its flexibility [4]. This method has been recently generalized to non-harmonic periodic perturbations achieving optimal suppression of chaos when the perturbation frequency is resonant with the main driving frequency [5]. Last, we present results when the perturbation is incommensurate with

respect to the main driving [6].

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## What can we learn from the spectra of quantum graphs and microwave networks?

Leszek Sirko, 1 Szymon Bauch, 1 Małgorzata Białous, 1 Vitalii Yunko, 1 Pavel Kurasov,<sup>2</sup> Jiri Lipovsky,<sup>3</sup> and Michał Ławniczak<sup>1</sup>

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We will discuss the role of spectra of quantum graphs and microwave networks in the revealing of their hidden properties. Using spectroscopy techniques we will demonstrate that there exist isoscattering graphs and networks [1-3]. We will prove that it is possible to construct graphs and networks which do not obey the Weyl's law  $N(R) = LR/\pi$ , where N(R) is the counting function of the number of resonances, k is the square root of energy, and L is the total length of a graph. Graphs which do not obey the Weyl's law are called non-Weyl graphs [4]. Finally, we will demonstrate that the spectra of microwave networks can be used as sensitive revealers of their topological properties allowing, e.g., for the identification of the number of independent cycles in the networks.

Keywords: Simulations, Quantum graphs, Microwave networks, Quantum chaos, Open systems, Microwave spectroscopy.

Acknowledgements: This work was supported in part by the National Science Centre Grant No. 2016/23/B/ST2/03979.

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### A Simplest 1-BJT-Based Chaotic Hyperjerk Circuit: Its Minimized Damping for Maximized Attractor Dimension, and Hidden Attractors

### Banlue Srisuchinwong

Sirindhorn International Institute of Technology, Thammasat University, Thailand

In high dimensional systems, a 4th-order time derivative, though no standard name, is often called snap, whereas a time derivative higher than the third is referred to as hyperjerk [1]. The presentation reviews a simplest chaotic hyperjerk, or snap for a single 4th-order ordinary differential equation (ODE), circuit based on a single bipolar-junction transistor (BJT) with tunable damping [2]. At present, it is the simplest chaotic hyperjerk circuit in the sense that the 4th-order ODE needs only 9 electronic devices, which appear to be the minimum number of devices for a chaotic hyperjerk circuit. It also offers the first and simplest circuit implementation of either a four-dimensional (4D) chaotic system or a 4th-order (snap or hyperjerk) chaotic system that exhibits, at minimized damping, a maximized attractor dimension (DL) of a parameter set, or of the full parameter space of the system. The increase trend in  $D_{\rm L}$  until its maximum is depicted by a decrease in damping. It provides the largest  $D_L$  in a category of unitdamping snap chaos. As an early report, a Clapp oscillator can demonstrate 4D chaos but does not allow hyperjerk chaos. The simplest hyperjerk circuit employs two simple techniques: (i) a Clapp oscillator is embedded as a simple core engine of oscillations avoiding a need for opamps, and (ii) a single resistor is embedded as a particularly simple implementation of tunable damping for hyperjerk chaos. A current-tunable equilibrium displays one of the 4 possible equilibria, two of which are of an (unstable) spiral saddle equilibrium, whereas the others are of a spiral stable equilibrium. They disclose the first report on either saddleequilibrium or stable-equilibrium snap chaos based on a single BJT. Hidden attractors and multistability are illustrated. The simple circuit demonstrates a new damping-tunable 1-BJT-based approach to rich dynamics of 4th-order hyperjerk flows through several self-excited and hidden attractors.

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## Variations on the Fermi-Pasta-Ulam chain, a survey

### **Ferdinand Verhulst**

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We will present a survey of low energy periodic Fermi-Pasta-Ulam chains with leading idea the "breaking of symmetry". The classical periodic FPUchain (equal masses for all particles) was analysed by Rink in 2001 with main conclusions that the normal form of the beta-chain is always integrable and that in many cases this also holds for the alfa-chain. The implication is that the KAM-theorem applies to the classical chain so that at low energy most orbits are located on invariant tori and display quasiperiodic behaviour. Most of the reasoning also applies to the FPU-chain with mixed endpoints.

The FPU-chain with alternating masses already shows a certain breaking of symmetry. Three exact families of periodic solutions can be identi--ed and a few exact invariant manifolds which are related to the results of Chechin et al. (1998-2005) on bushes of periodic solutions. An alternating chain of 2n particles is present as sub-manifold in chains with k 2n particles, k=2, 3, ... The normal forms are strongly dependent on the alternating masses 1, m, 1, m,... If m is not equal to 2 or 4/3 the cubic normal form of the Hamiltonian vanishes. For alfa-chains there are some open questions regarding the integrability of the normal forms if m= 2 or 4/3. Interaction between the optical and acoustical group in the case of large mass m is demonstrated.

The part played by resonance suggests the role of the mass ratios. It turns out that in the case of 4 particles there are 3 first order resonances and 10 second order ones; the 1:1:1:...:1 resonance does not arise for any number of particles and mass ratios. An interesting case is the 1:2:3 resonance that produces after a Hamilton-Hopf bifurcation and breaking symmetry chaotic behaviour in the sense of Shilnikov-Devaney. Another interesting case is the 1:2:4 resonance. As expected the analysis of various cases has a significant impact on recurrence phenomena; this will be illustrated by numerical results.

**Keywords:** FPU-chain, resonance, periodic solutions, normalisation, chaos, Hamilton-Hopf bifurcation.

# **Invited and Contributed Talks**

## **Control of Dissipative Dynamical Systems**

### Meryem Abaakil

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In this work, we propose a new feedback control for the control of general dissipative dynamical systems through a variety of determining parameters (modes, nodes, finite volume elements, etc.) without requiring the presence of a spatial separation, that is to say without assuming the existence of an inertial variety. To set ideas, we prove our idea on a simple reaction diffusion equation, the Chafee-Infante equation, which is the real form of the Ginzburg-Landau equation. In addition, we would like to emphasize that our system even considers all kinds of determinig projections as well as the various dissipative equations with a united approach. However, this new idea has much richer applications other than the control of feedback, as in the data assimilation for meteorological forecasting. Our study is going on a control by feedback of finite dimension to stabilize the stationary solutions of the dissipative dynamical systems of infinite dimension. Thanks to this kind of control the system stabilizes under specific conditions, which can thus be expressed in the form of linear matrix inequality (LMI). The aim is to suggest a new nonlinear stabilization approach, for the first time according to the best available knowledge, by adapting a method combining linear matrix inequalities and the Lyapunov-Krasovskii's theorem used for the stabilization of delay

Keywords: Feedback Control, Dissipative Dynamical Systems, Nonlinear Dynamics, Interpolant Operators, Linear Matrix Inequalities, Lyapunov Krasovskii's Theorem.

# Higgs boson and Higgs field in fractal models of the Universe: active femtoobjects, new Hubble constants, solar wind, heliopause

### Valeriy S. Abramov

Donetsk Institute for Physics and Engineering named after A.A. Galkin, Donetsk, Ukraine

Theoretically the relationship between the main parameters of active femtoobjects and the Higgs boson in fractal models of the Universe was investigated. To describe the structure of the solar wind, heliopause, new Hubble constants are proposed. Estimates of the main parameters are conformed with the experimental data obtained by the Planck space observatory (based on Fermi-LAT and Cerenkov telescopes), UTR-2 and URAN-2 radio telescopes, Parker Solar Probe, Voyager 2 and Voyager 1. Within the framework of the anisotropic model, a description of the main characteristics of the model femtoobject and its relationships with the parameters of the Higgs boson and the Higgs field was performed. To take into account the stochastic behavior of the parameters of a model femtoobject (an active object with dimensions of the order of the classical electron radius), random variables are introduced. Using the example of a hydrogen atom, we estimated the radius of a proton, its mean square deviation, and compared it with an experiment. Estimates of the anomalous contributions to the magnetic moments of leptons based on the lepton quantum number are obtained.

Keywords: model femtoobject, Higgs boson and Higgs field, fractal models of the Universe. Hubble constants, structure of the solar wind. heliopause, hydrogen atom, proton and electron radii, magnetic moments of leptons.

# Qubits and fractal structures with elements of the cylindrical type

Olga P. Abramova, Andrii V. Abramov Donetsk National University, Donetsk, Ukraine

By the method of numerical simulation, the behavior of the deformation field of both separated and related model fractal structures of a cylindrical type was investigated. It is shown, that for the considered structures, the behavior of the deformation field essentially depends on the choice of stochastic processes (realized during iterations) and on the states of the qubit in the perpendicular plane to the axis of the cylinder. It is shown that the structure of the complex deformation field for a circular (elliptical) cylinder essentially depends on the initial basic, superposition states of the qubit. Due to the presence of various qubit states for coupled (using the example of circular and elliptic cylinders) fractal structures, the appearance of random matrices during iterations is characteristic. There is a need to use commutators and anti-commutators, products of separate deformation field opera! tors. At this, the structure of the complex deformation field has own characteristic features of behavior.

**Keywords:** fractal structure, qubits, random matrices, complex deformation field, ordering of operators, quantum chaos.

# Fractal atomicity, a fundamental concept in the dynamics of complex systems

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Applying a multifractal method of analyzing the dynamics of the structural units of any complex system, a mathematical concept is built, namely that of fractal atomicity. The construction of such a concept involves defining dynamic variables in the form of multifractal functions, defining scale resolutions, defining a principle of scale covariance as a fundamental principle of motion, equations of evolution, etc. Finally, some specific mathematical properties of the holographic atom as well as an application from the dynamics of biological structures in the form of the LDL-HDL state transition are presented. Several implications in the study of brain dynamics are also provided.

**Keywords:** Fractal atomicity, Complex systems dynamics, Multifractal functions, Holographic atom.

# Statistical methods and nonlinear dynamics for analyzing brain activity. Theoretical and experimental aspects

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In this paper, we propose statistical methods and nonlinear dynamics for analyzing brain activity in epileptic patients, using the PhysioNet database. Thus, the analysis by statistical methods (the time variation of standard deviation of the component signals electroencephalogram, the time variation of the signal variance, the time variation of the skewness, the time variation of the kurtosis, the construction of the recurrence maps corresponding to both normal functioning of the brain, as well as of the pre-crisis period, respectively of the crisis, the evolution in time of the spatial-temporal entropy, the variations of the Lyapunov coefficients, etc.) allows us to determine not only the epilepsy time based on a specific strange attractor but also that the entry into the epileptic seizure can be determined at least twenty minutes in advance. Finally, a fractal model is built based on a specific operational procedure (multifractal analysis, group invariants, geometric probabilities based on integral geometry, etc.). The validation of the theoretical model was achieved both by comparing theoretical and experimental signals, as well as comparing the theoretical strange attractors and the experimental ones.

**Keywords:** Brain activity, Epilepsy, Electroencephalogram, Fractal model, Signal, Skewness, Kurtosis.

# On Approximate Conformal Mappings for Domains with Fractal Boundaries

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In the report presented we discuss construction of approximate conformal mapping of the nearly circular domain on unit disk. A feature of our approach is taking into account a fractality of domain boundary. In order to describe this property of the boundary we use the Weierstrass function. To provide  $2\pi$ -periodicity of this function over polar angle one ought to choose parameter b of this function as even number. Further in accordance with methods developed in the middle of the 20th century by the soviet mathematicians M.A. Lavrentyev [1] and G.V. Siryk [2] for derivation of the desired mapping the boundary must be guite smooth. In order to overcome this obstacle in correspondence with suggested by A.A. Potapov concept of physical fractals (see [3] and references therein) we truncate the functional series representing the Weierstrass function. But of course this truncated series ought to contain enough terms to approximate the input function quite exactly. The aim of this research work is to estimate the influence of fractal roughness of boundary for round cylindrical hole in conductor on distribution in it of electrostatic potential and electric field strength in the framework of perturbation theory using as starting point the well-known modification of the Poisson integral.

**Keywords:** The Weierstrass function, Physical fractals, The Dirichlet problem, Perturbation theory, Electrostatic potential, Electric field strength.

**Acknowledgement:** This work was supported by the Russian Foundation for Basic Research, project No. 18-08-01356-A.

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## Localized waves in silicates. What we know from experiments?

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Since the latest review about solitary localized waves in muscovite, called auodons, IFM Russell, Springer Ser, Mater Sci. 221 (2015) 31 there have been many developments, specially from the point of view of experiments. published in several journals. The breakthrough hypothesis that was advanced in that review that dark tracks were produced by positive electrical charge moving in a localized wave, either transported by swift particles or by nonlinear localized waves, has been confirmed by experiments in muscovite and other silicates. In this paper we review the experimental results, some already published and some new, specially the phenomenon of charge transport without an electric field, called hyperconductivity. We also consider alternative explanations as phase transitions for other tracks. We also describe numerical simulations that have confirmed the order of magnitude of quodons energy and calculations underway to determine more properties of electron and hole transport by quodons.

Keywords: Layered silicates, nonlinear waves, quodons, kinks, breathers, charge transport, hyperconductivity.

# Hopf bifurcation analysis for the Fitzhugh-Nagumo model of a spiking neuron

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Neurons, the smallest members of the brain, transmit information between each other through electrical activities, which can be modeled and analyzed by dynamical systems. This communication among neurons observed as firing or spikes occurs as an oscillation formation or loss around a singular point, i.e., Hopf bifurcation when a parameter of the dynamical system exceeds a threshold value. Because of the complexity of the neural dynamical models consisting of multi-parameter nonlinear systems, often results are given only in a particular situation by a numerical solution and general results cannot be obtained. In this study, we employ methods of dynamical systems together with computational

algebra for the dynamical analysis of the Fitzhugh-Nagumo model, and we present parameter regions for the existence and the stability of limit cycles for the Fitzhugh-Nagumo model. The Fitzhugh-Nagumo model, which was proposed by R. Fitzhugh in 1961 and simulated by J. Nagumo et al. in 1962, is governed by the ODE system

$$\dot{x} = x(x - a)(1 - x) - y$$
$$\dot{y} = \epsilon(x - \gamma y)$$

where state variables x(t) and y(t) represent the change in the membrane voltage (called "action potential") and the change in the number of open potassium channels, respectively, over time. The Fitzhugh-Nagumo model, which describes a pulse transmission activity in a neuron, is first called the Bonhoeffer-van der Pol model since it is originally transformed from the well-known van der Pol model. The analysis of limit cycles of the Fitzhugh-Nagumo model, which mathematically express the spiking event that occurs in the communication of neurons of this model, will contribute to the understanding of the working principle of the neural system and give a hint about how well such models describe the real phenomenon.

**Keywords:** Fitzhugh-Nagumo model, limit cycle, stability, periodic solution.

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# Limit cycles of the Schnakenberg chemical reaction model

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Dynamical systems contribute to mathematical modeling of chemical reactions of biological or ecological phenomena producing limit cycle oscillations. In this study, we present a computational approach to

examine the bifurcations of limit cycles of the following two-component simple chemical reaction model known as the Schnakenberg model.

$$\frac{dx}{dt} = x^2y - x + b,$$
$$\frac{dy}{dt} = -x^2y + a$$

With our approach, we obtain parameter regions for oscillatory solutions of the chemical model which gives Hopf bifurcation. We calculate the first Lyapunov coefficient to show the stability of Hopf bifurcation. We illustrate the results with numerical simulation.

**Keywords:** Schnakenberg, Limit cycle, stability, Lyapunov coefficient, chemical reaction.

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# On the Dirichlet Problem with Fractal Boundary Condition

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In this work we discuss the two-dimensional Dirichlet problem on halfplane. A peculiarity of our point of view on the problem is in consideration of fractality of its boundary condition. As a model of fractal boundary condition we take the well-known Weierstrass function because of one can vary its fractal dimension by means of changing of its parameters. We construct exact solution u(x, y) of the problem using the Poisson integral for upper half-plane [1]. From the de Rham functional equation for the Weierstrass function (see [2] and references therein) it is easy to find that this exact equation u(x, y) obeys to some functional equation expressing its self-similarity too. If one interpret function u(x, y) as electrostatic potential then the results obtained might be useful for explanation of fractal structure of elves, jets and sprites established in [3] in the framework of fractal image processing.

**Keywords:** The Poisson integral, Fractal dimension, the Weierstrass function, de Rham functional equation, Self-similarity, Electrostatic potential.

**Acknowledgement:** This work was supported by the Russian Foundation for Basic Research, project No. 18-08-01356-A.

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# A New Substitution Box Structure Based on Nose– Hoover Chaotic System

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A cryptographic protocol should provide two basic requirements for secure communication. These requirements are known as diffusion and confusion. Substitution box structures are needed in order to provide the confusion requirement in block encryption algorithms. These cryptographic blocks must have a nonlinear structure to meet the confusion requirement. Various designs based on chaotic systems have been proposed to ensure the nonlinearity requirement. In this study, a new substitution box structure based on Nose–Hoover Chaotic System is proposed. Successful analysis results showed that the proposed new chaos based substitution box structure could be an alternative to the other three degree chaos based substitution box structures. An implementation of this substitution box structure is illustrated on image encryption algorithms.

**Keywords:** chaos, cryptography, substitution box, image encryption.

# On the comparison of Liutex method with other vortex identification methods in a confined tip-leakage cavitating flow

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In the current paper, Large Eddy Simulation on the confined tip leakage cavitating flow generated by a straight NACA0009 hydrofoil with a clearance size gap=10mm is performed. TLV cavitation and TSV cavitation are reasonably captured, which evidences a good agreement between the simulation results and experimental data. Three stages of the spatial development of tip vortices are investigated in detail with vortex identification methods of all three generations, including vorticity, Qcriterion, 
method and Liutex method. It is observed that, with a clear physical meaning as the rigid rotation part of fluid motion. Liutex method shows advantages in visualizing vortical structures in the currently studied flow regime. A strong shear layer is found around TLV, which plays an important role in the evolution of tip vortices.

**Keywords:** Vortex identification, LES, Vortex cavitating flow, Tip-leakage flow, Liutex method.

## Shilnikov Chaos, Low Interest Rates, and New **Keynesian Macroeconomics**

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The paper shows that in a New Keynesian (NK) model, an active interest rate feedback monetary policy, when combined with a Ricardian passive fiscal policy, à la Leeper-Woodford, may induce the onset of a Shilnikov chaotic attractor in the region of the parameter space where uniqueness of the equilibrium prevails locally. Implications, ranging from long-term unpredictability to global indeterminacy, are discussed in the paper. We find that throughout the attractor, the economy lingers in particular regions, within which the emerging aperiodic dynamics tend to evolve for a long time around lower-than-targeted inflation and nominal interest rates. This can be interpreted as a liquidity trap phenomenon, produced by the existence of a chaotic attractor, and not by the influence of an unintended steady state or the Central Bank's intentional choice of a steady state nominal interest rate at its lower bound. In addit! ion, our finding of Shilnikov chaos can provide an alternative explanation for the controversial "loanable funds" over-saving theory, which seeks to explain why interest rates and, to a lesser extent inflation rates, have declined to current low levels, such that the real rate of interest is below the marginal product of capital. Paradoxically, an active interest rate feedback policy can cause nominal interest rates, inflation rates, and real interest rates unintentionally to drift downwards within a Shilnikov attractor set. Policy options to eliminate or control the chaotic dynamics are developed.

**Keywords:** Shilnikov chaos criterion, global indeterminacy, long-term unpredictability, liquidity trap.

Chaos and Global Indeterminacy in Dynamic Economic Models (Chair Prof. Beatrice Venturi).

## Control of a Modified Chua's System using the Routh-Hurwitz Criteria

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The aim of this paper is to study an alternative and adaptive controller for resolving the control problem of the modified chua's circuit system; our work is based on Menacer et al (2016) paper. It is assumed that the parameter  $\epsilon$  is only and the only one state variable, is governing the feedback controlled. We use the Routh-Hurwitz Criteria to study the stability of autonomous chaotic system. Numerical results show the effectiveness of the theoretical analysis.

**Keywords:** Chaos Control, Routh-Hurwitz Criteria, Modified Chua's system.

# Selective transport of airborne microparticles in a microgravity environment

### **Philippe Beltrame**

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This paper analyzes the possibility to obtain selective transport of microparticles suspended in the air in microgravity environment through modulated channels without net displacement of air. Using numerical simulation and bifurcation analysis tools, we show the existence of

intermittent particle drift under the Stokes assumption of the fluid flow. For specific parameter ranges, the particle transport can be selective and the direction of transport is controlled only by the kind of pumping used. The selective transport phenomenon belongs to deterministic ratchets due to non-linear dynamics. We reveal that chaotic transitions are a key factor to drop from a diffusive behavior to a net transport. This ratchet phenomenon can be checked with metal particles during the short duration of microgravity experiments.

**Keywords:** Chaotic dynamics, transport, intermittency, Bifurcations, microparticle sorting, Stokes flow.

# Inhibited-Coupling guiding hollow core PCF: A platform for pulse compression

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The capability of inhibited-coupling guiding hollow-core photonic crystal fibers (IC-HCPCF) in guiding ultra-high energy ultra-short pulses with ultra-low transmission loss and with controllable dispersion enabled demonstration of varied and strong pulse compressions, and commercial compression modules based on IC-HCPCF technology are now available. We review different results obtained on ultra-short pulse compression including compression down to the single cycle, compression at wavelength range in the ultra-violet and infrared, or with milli-Joule energy level. We list the different pulse compression mechanisms encountered in gas-filled IC-HCPCF, which includes self-phase modulation, soliton self-compression or plasma-initiated soliton formation.

**Keywords:** hollow-core photonic crystal fiber, nonlinear optics, pulse compression.

# Point Cloud Wave Attractor from Discrete Rotation-Translation Sequences on Closed Loops

### **Bernd Binder**

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Consider an ordered sequence of operations given by a rotation, a translation, and a reflection. Repeating this sequence operating on an initial discrete point cloud provides for characteristic spatial density

patterns in the plane. A wide variety of interesting chaotic point clouds emerge if the rotation angle is a monotonic decreasing function of distance from a rotation singularity. In this way C. Skiadas generated complex structures resembling familiar chaotic flow patterns. Adding a periodic boundary condition, wave-like patterns can emerge (B. Binder) on the sphere and in the plane. We focus on closed loop conditions, where the point cloud evolves toward a sine wave attractor independent of initial conditions. As a key boundary condition, the rotation-translation loop sequence generates a highly nonlinear extra geometric phase gap controlling the pattern type with chaotic patterns at stronger rotations. Reversing the order of operations computes the gap as a difference in the step by step propagation as a nonzero rotation-translation commutation or time reversal in the operator sequence.

The emerging waves can always be characterized by

- an integral wave number,
- a geometric phase shift or gap with respect to the singularity location,
- a characteristic number of discrete operations or steps completing one loop. Finally, we propose that this computer experiments show some relevance to quantum physical systems since we have
- a wave attractor showing quantization effects in terms of rotational units,
- a basic non-zero commutation relation in the two main operators (rotation-translation).
- point-like local events with emerging global wave-like probabilistic patterns.

**Keywords:** Chaotic modeling, Rotation, Discrete, Translation, Reflection, Closed Loop, Quantum, Chaotic attractors, Patterns, Simulation, Commutation, Operator, Geometric Phase.

# Tilted Nadaraya-Watson Nonparametric Regression Estimator

# Farzaneh Boroumand<sup>1</sup>, Hassan Doosti<sup>2</sup>, Mohammad Taghi Shakeri<sup>3</sup>, Nino kordzakhia<sup>4</sup>, Mehdi Salehi<sup>5</sup>

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Tilting techniques are sustainably used for adjusting the empirical distribution by replacing uniform distribution of weights over data by general multinomial distribution. The tilting approach has been used for minimizing the distance to an infinite order regression estimator. We proved that the tilted Nadaraya-Watson estimator achieves a higher level of accuracy at the same time preserving the attractive properties of infinite order estimator. We also showed that the tilted estimators are consistent and have optimal convergence rates. In a simulation study, we illustrated that the tilted version of the Nadaraya-Watson estimator performs better than its classical analog in terms of Median Integrated Squared Error (MISE).

**Keywords:** Curve smoothing, Kernel, Nonparametric regression.

# Analysis of Bifurcations in a Model of Coupled Josephson Junctions: Some Challenges for Standard Numerical Continuation Methods

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When an external dc-current passes through a stack of coupled Josephson junctions, the system is in a nonequilibrium state and there is a diffusion contribution to the quasiparticle current, due to the generalised scalar potential difference between superconducting layers. The inclusion of the diffusion current (DC) into phenomenological nonlinear dynamic models of coupled Josephson junctions (the so-called CCJJ+DC model). leads to a better agreement between the theoretically predicted and experimentally observed IV-characteristics~cite {shu06}. Additionally, it permits several new resonance features which do not exists in the case of a single junction. Particularly, the possibility of a double resonance occurs when the Josephson frequency is commensurate with that of externally applied electromagnetic radiation and the longitudinal plasma wave that gets excited along the c-axis of the stack~cite{koyama96}. Similarly a triple resonances is also possible when the system is shunted by LCRcircuit elements, which have their own natural resonance frequency. Although some of the physical and even technological applications of the additional diffusion current have already been explored through detailed numerical simulations~cite {shu17}, there have been no proper

mathematical analyses of the same system. In particular, the bifurcations that occur as the coupled system is brought into and out of resonance have not been analysed. Even for the case of single junction, which can be adequately described by the simple RCSJ model~cite {noe86}, the application of standard numerical continuation methods to analyse the bifurcations is complicated by long transient states and convergence to undesired solutions~cite {san14}. In the present work we will therefore analyse the bifurcations that occur in the coupled model of Josephson junctions via standard numerical continuation and other methods.

Title unknown, Prof Yury Shukrinov.

Keywords: Josephson Junctions.

# Application of New 4-D Chaotic Map for Secure IP-Communication

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Chaos systems have been studied for decades due to their applications such in several domains as: economy, communications. cryptography...etc. In recent years, designing and proposing new and higher dimensional chaotic systems become an increased tendency in particular for chaotic systems applied in network security domain. In this paper, we propose a new 4-dimension chaotic map with four (04) control parameters and seven (07) non-linear terms. Then, we investigate the chaotic behaviors of the proposed system by considering the bifurcation and the Lyapunov exponents (LE) theories. Finally, the proposed map is applied for generating cipher keys to perform data encryption and secure an Internet Protocol (IP) communication.

**Keywords:** Chaotic, Dimension, Bifurcation, Lyapunov exponents, Network security, Encryption, IP-communication.

# Experimental Study on vortex structures in turbulent boundary layer by Liutex identification

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Multilevel vortex structures in a near-wall turbulent boundary layer is experimentally investigated by a motion single frame and multiple exposure imaging method. In order to study the process of the vortex evolution, the measuring system moves at a constant speed to track the vortex structure with a similar speed. The Liutex (vector), which is a new vortex core identification method proposed by Liu's group, is utilized for the structure visualization and quantitative analysis of the local fluid rotation in our experiment. In this study, on the one hand, a solid evidence of how energy can be transformed from the main flow to the bottom of the boundary layer is proposed. On the other hand, the mechanism of the vortex generation and evolution is given through the Liutex vector field analysis. Moreover, the results give four stages of the vortex merging process, and also reveal the criteria of the vortex merging among multiple vortices (two or three) in terms of their sizes, distance, and direction and intensity of rotation or shear.

Keywords: Turbulence, Boundary Layers, Liutex, Experiment.

# Exploring the spatiotemporal dynamics in active nanocompartment systems

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Oscillatory chemical reactions carried out in water-in-oil microemulsions have been thoroughly studied during last decades as a paradigm of nonlinear dynamical systems. This chemical example displays a large amount of different dynamics such as stationary Turing structures, standing waves, bulk oscillations, jumping waves or spatiotemporal chaos, just to mention a few. These dynamical patterns have been experimentally proven that depend on both the chemical kinetics and the microemulsion features. Among the dimensionless reaction-diffusion models, this work explores a three variable extended FitzHugh-Nagumo model to mimic those chemical steps occurring in these active nanocompartments. explaining the wealth of spatiotemporal dynamics in terms of different diffusive instabilities. This system is composed by a fast inhibitor diffusing through the entire microemulsion system while a slow activator and inhibitor are confined inside the aqueous nanodroplets with a diffusion coefficient dependent on the volume droplet fraction

**Keywords:** Chaos and nonlinear dynamical systems, chemical chaos, FitzHugh-Nagumo model, chaotic simulation, pattern formation, diffusive instabilities.

# Generation of surrogates of spike-like chaotic sequences

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Many physical systems, in context ranging from neuroscience to geophysics and economics, produce signals characterized by sequences of events, for which information is encoded in the sequence of interevent intervals. The application of analytical techniques to these sequences, such as the evaluation of nonlinear properties or the degree of synchronization between multiple signals, leads to the crucial issue of assessing the statistical significance of the outcomes. Because the nullhypothesis distribution is often unknown, an alternative solution consists in inferring significance directly from the available data: the same analytical techniques are applied to randomly-generated surrogate data that share statistical and spectral properties with the original dataset. Conventional methods for surrogate generation, despite being efficient in the case of continuous signals, become computationally demanding when applied to event sequences. Our recently developed method, which relies on the joint distribution of interevent intervals, overcomes previous methods in terms of computational speed while providing the same degree of reliability. In this presentation, we apply this new method to spike-like signals generated by chaotic attractors, as well as on sequences coming from chaotic maps.

**Keywords:** Interevent Intervals, Surrogates, Time Series Analysis, Joint Distribution, Chaotic attractors, Chaotic Maps.

Dynamic Mode Decomposition of Liutex to Identify Vortices in Early Flow Transition

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There have been many methods widely used as vortex identification methods to identify vortices such as vorticity tubes/lines, Q-criterion and λ 2-criverion. However, these existing methods fail to obtain the rotational axis and strength correctly. Moreover, many researchers have proved that the existing methods are not the purely rotational motions, but contaminated by shears or stretches. Hence, it is not appropriate to apply them for vortex identifications since vortices are recognized as rotations of fluid flows. Recently, a new method named Liutex has been introduced by Liu et al. (2019) to effectively identify the rotational parts of fluid flows. Also, a Liutex vector is defined as a local rotational axis with the Liutex magnitude, which is defined as the rotation strength. In this work, Liutex is applied to analyze the coherent structures of rotations with the dynamic mode decomposition in the early flow transition on the flat plate by DNS data. Since the dynamic mode decomposition can extract the single frequency and amplitude, it can be investigated the relation between frequency, amplitude and Liutex magnitude. The results present that the correlation between frequency and Liutex magnitude is highly significant. Moreover, this analysis shows that the large scale of streamwise vortices are captured by the lowest-frequency mode, i.e. highest Liutex magnitude. For the hairpin vortex structures, they can be captured by only the few low-frequency modes, i.e. high Liutex magnitudes.

Keywords: DMD, Vortex, Flow Transition, Liutex.

## Numerical investigation of the cavitation vortex interaction around a twisted hydrofoil with emphasis on the vortex identification method

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The four kinds of vortex identification method, namely, ω criterion, Q criterion,  $\Omega$  method and Liutex method, is then employed to predict vortex structures of unsteady cavitating around a twisted hydrofoil. The results show that the cavity-vortex structure of the primary shedding process is quite complicate, which contains U-type structure, 3D waves of the vortex structures and large-scale vortex structures. For U-type structure, it should be noted that compared to other vortex identification method, Liutex method can not only clearly display the U-type vortex structure, but also

accurately describe its direction, and the direction of U-type vortex structure by the Liutex vector line is counter-clockwise. For 3D waves of the vortex structure, located in non-cavitation zone on the surface of hydrofoil, which is accurately captured with other vortex identification method, except for the  $\omega$  criterion. For large-scale vortex structures near the TE of hydrofoil, induced by the cavity shedding, which is clearly presented by Q criterion,  $\Omega$  method and Liutex method, whereas the  $\omega$ criterion only present the small-scale shedding vortices in the same position. Based on above description and discussion, Liutex method has a relatively superiority for other vortex identification method to accurately present both vortex magnitude and the direction of vortices well simultaneously, especially for the direction of vortex. Therefore, Liutex method is further applied to visualize the detail of vortex structure in the shedding process, and it is found that the U-type vortex structure is the main feature of the shedding process, accompanied by other forms of vortex structure, such as the hairpin vortex and the O-type vortex structure

**Keywords:** Numerical analysis, Cavitating flows, Vortex structure, Liutex method, U-type vortex.

### Application Modified Liutex-Omega method to High-Temperature Supersonic Turbulent Channel Flows

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Vortex structures is one of important characteristics in the turbulent flows, whose are often studied by the vortex identification and visualization. For supersonic turbulent flows, the temperature near the wall may be very high and the assumption of calorically perfect gas is no longer applicable. In the present study, based on the DNS results of high-temperature supersonic turbulent channel flows, modified Liutex-Omega method is introduced to investigate the characteristics of vortex structures. Moreover, the effects of gas model, including thermally perfect gas and equilibrium air, on the vortex structures are also studied.

**Keywords:** Liutex, Direct numerical simulation, Channel flow, High-temperature effect, Supersonic flow.

### A comparison of Liutex with other vortex identification methods on the multiphase flow past a cylinder using LBM on GPU

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The multiphase flow involves a series of prevalent phenomena that occur in massive industrial and natural processes. The flow past obstacles with extensive application in engineering has aroused wide research attention. Due to the complexity of interaction between different phases coupled with the impact of obstacle on the fluid, the various flow patterns and the underlying mechanism require further exploration in numerical simulation. The Lattice Boltzmann method (LBM) has developed rapidly in the past decades especially in simulation of multiphase flows. As a novel mesoscopic approach, it enjoys the advantage of natural parallelism, flexible geometry characteristics and simplicity of implementation. The widely adopted Shan-Chen multiphase model provides a simple and efficient method for multiphase flow and has achieved ongoing progress in diverse application cases such as boiling and condensation. Thanks to the inherent parallelism in LBM algorithm, the parallel implementation of LBM on GPU has remarkably reduced the computation time and cost, making it an ideal choice. As a fundamental research target, the vortex has significant meaning for an intuitive understanding of turbulence. After decades of intensive research, there are still plenty of ambiguous issues left to be solved. The vortex identification method has evolved from the first generation, namely vorticity-based method, to the second generation that rely on other parameters including Q,  $\lambda$ ci,  $\Delta$ , and recently to the third generation Liutex/Rortex method proposed by Liu. The Liutex method decomposes vorticity into a rotational part R representing the rigid-body rotation and a non-rotational part S related to shear, which has tackled the problem that vorticity doesn't sufficiently represent rotation. In this paper we will focus on the flow field of multiphase flow past a 2D cylinder based on results from LBM on GPU. Several influential vortex identification methods are selected including the Liutex. The mechanism of change in variables especially Liutex along with the movement and deformation of drops as well as collision with the obstacle is analyzed. The performance of different methods in characterizing rotation in flow is also compared. Keywords: Multiphase flow modeling, The Lattice Boltzmann method

(LBM), GPU, Vortex identification method, Liutex/Rortex method.

Image based methods to investigate causality between time series relevant for plasma fusion diagnostics

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Advanced time series analysis and causality detection techniques have been successfully applied to the assessment of synchronization experiments in tokamaks, such as ELM and sawtooth pacing [1, 2]. Lag or intermittent lag synchronization, where the difference between the output of one system and the time-delayed output of a second system are asymptotically bounded, is a typical strategy for fusion plasma instability control by pace-making techniques. The major difficulty, in evaluating the efficiency of the pacing methods, is the coexistence of the causal effects with the periodic or quasi-periodic nature of the plasma instabilities. In this context, in the present work, a set of methods based on the image representation of time series, are evaluated as tools for unravelling the cause-effect relations. The Gramian Angular Field (GAF) and the Markov Transition Field (MTF), previously used mainly for time series classification, exploit techniques from computer vision for time series analysis [3]. GAF images uses a polar representation of time series and each element is derived by means of a trigonometric relation between time intervals. MTF images represent the first order Markov transition probability along one dimension and temporal dependency along the other. The Chaos Game Representation (CGR), inspired by a drawing fractals algorithm [4], allows the representation of long one-dimensional sequences as images for visualisation and identification of hidden structures. For causality analysis, the relations between time series, represented as images, are analysed by mean of the Structural Similarity (SSIM) index [5], originally developed as a method for predicting the perceived image quality. Recently we proposed a method for characterization of interconnected dynamical systems coupling based on the representation of time series as weighted cross-visibility networks [6]. The structure of the networks may also be represented as an image. The structure evolution is quantified via the entropy of the image representation. Besides entropy, the efficacy of other quantifiers is investigated. The performances of the method are evaluated on synthetic data and applied to JET data.

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# Sequence of random wave excitations and minimum entropy production in tokamak plasma

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We analyse fluctuations around the minimal excitation regimes of typical instabilities in tokamak (ITG and trapped electron). We formulate a model of "k-space avalanches" based on the inverse cascade that is supported by the cuasi-two-dimensional geometry and the dominantly fluid behavior of tokamak plasma. We show that the model can be mapped onto the Kardar-Parisi-Zhang (KPZ) statistics. We examine the possible application of the two-loop renormalization of the latter in the calculation of the correlations of fluctuations of the turbulent field in plasma. We also discuss the possible formulation in terms of the wave kinetic equation. For this to be an adequate description we need to adopt ad-hoc terms depending on the gradient of the flux in k-space, similar to the driving terms of KPZ. The "k-space avalanches" being of the nature of self-organized criticality in k-space. we consider the relevance of the minimum rate of entropy production. This depends on the saturation and dissipation of the cuasi-coherent potential structures (convective cells) that are randomly generated by cascading k-space pulses.

# Rigorous Scattering for Spheres of Arbitrary Size. A non-Linear Sequence

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All problems of theoretical optics are problems in Maxwell's theory and should be treated as such when a full, formal solution is required. The scattering of light by a homogeneous sphere cannot be treatment in a general way, other than by the formal solution of Maxwell's equations with the appropriate boundary conditions.

Mie's Formal Solution

Spherical coordinates of a point P will be denoted by  $(r, \theta, \phi)$  with the usual meaning of symbols. Mie's problem: scattering of a plane wave by a homogeneous sphere. In order to simplify the notation, we assume that the outside medium is vacuum ( $m_2 = 1$ ). The material of the sphere has an arbitrary refractive index m. We assume that the incident radiation is linearly polarized. The origin is taken at the center of the sphere, the positive z- axis along the direction of propagation of the incident wave. and the x-axis in the plane of electric vibration of the incident wave. For spheres of arbitrary size, the Born series for each excited electric or magnetic multipole has a complicated structure, but one that is shown to be exactly equivalent to the celebrated Mie results. A significant advantage over the usual differential approach is the implicit inclusion of the electromagnetic boundary conditions at the scattering surface in the integral approach, a feature that may provide valuable insights into scattering from arbitrarily shaped particles. The Mie solution to Maxwell's equations (also known as the Lorenz-Mie solution, the Lorenz-Mie-Debye solution or Mie scattering) describes the scattering of an electromagnetic plane wave by a homogeneous sphere. The solution takes the form of an infinite series of spherical multipole partial waves. Mie scattering (sometimes referred to as a non-molecular or aerosol particle scattering) takes place in the lower 4.5 km of the atmosphere, where there may be many essentially spherical particles present with diameters approximately equal to the size of the wavelength of the incident light. Mie scattering theory has no upper size limitation, and converges to the limit of geometric optics for large particles.

Dissipative solitons stabilized by nonlinear gradients

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We present a novel stabilization mechanism for dissipative solitons in the cubic complex Ginzburg-Landau equation with a nonlinear gradient term. We focus in particular on the influence of the Raman term. We make use of a mechanical analog containing a contribution from a potential and from a nonlinear viscous term. We show that the stabilization mechanism relies on the continuous supply of energy as well as on dissipation of the stable pulse. A quintic contribution is not necessary for stabilization in the presence of a suitable nonlinear gradient term as we show using a linear stability analysis for the stationary pulses. We find that the limit of vanishing magnitude of the nonlinear gradient term is singular: for exactly vanishing nonlinear gradient term stable pulses do not exist. We demonstrate that for small magnitude of the nonlinear gradient term simple types of scaling behavior are found for the amplitude, the full width at half maximum (FWHM), the velocity and the effective frequency of the stable pulse as a function of the magnitude of the nonlinear gradient term. **Keywords**: Dissipative soliton, Complex Ginzburg-Landau equation.

### Systemic effects of the credit crunch. A dynamic mathematical model

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This paper began with the hypothesis that there is an intrinsic mutualistic dependence between the bio-economic performance of banks and that of enterprises. Indeed this supposition is sustained by the comprehensive presentation herein of the correlations calculated on the data of Italian system. At the very least, this law seems plausible in relations among financial intermediaries and microenterprises and very small businesses. Therefore, this research concentrated on developments within the bankenterprise system (and, as a repercussion, in households), since clearly, the positive effects, including the impact on macroeconomics, generated by an efficient banking sector in the supply of funding to the productive infrastructure, so that it can remain healthy and efficient, should not be overlooked (nor, on the contrary, should the negative effects produced by the disappearance such a virtuous cycle). Hence, through the use of a dynamic model, this paper posits a mathematical argument, which evaluates the structural trends of the populations of banks and companies with respect to the more or less expansive strategies of the former, in granting credit. Empirical observations of the negotiation of loans to enterprises can also be represented as a "critical stress utilization" of the capacity of (micro) enterprises to generate positive economic variations using ever less financial leverage. This has led to the assessment of stable and unstable points of equilibrium, of the bifurcations generated and of their irreversibility (hysteresis), resulting in, on the banking side, stopping profits and an increase in non-performing loans.

**Keywords:** dynamic modelling, simulation, credit big data, economic systems, financial stability.

### Furstenberg family with IP set in Distributional Chaos

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In this work we going to study the properties of DC systems using families generated by IPsets. We got results studying the properties of a Distributional chaotic system of type 1 and 2 in block families that contains IP-sets and has piece-wise syndetic sets and positive upper Banach density sets. Other result is that a Special type of family in scattering system with Proximal relation has DC2, since a Weakly mixing and it has Sensitive Family, which meets elements with a dual space of the Special Family.

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# Double Layer in Plasma as an Interface Generated through the Interaction of Two Fractal Fluids

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Double layers are the well-known nonlinear potential structures in plasma. consisting of two adjacent layers of positive and negative space charges, sustaining a potential jump. A double layer can exist in either a static or dynamic state, the later one consisting of periodic disruptions and reaggregations of the structure. A linear dependence between the dynamics' frequency and the dc component of the current through the double layer was experimentally observed. Supposing that, due to the collisions, the motion of the plasma particles (electrons, ions, neutrals) take place on continuous but nondifferentiable curves (fractal curves), a theoretical model was developed, able to describe the dynamic behavior of the double layers. In this model, the double layer appears as narrow interface generated through the interaction of two fractal fluids. The dynamic state of the double layer develops when a constant voltage (up to a critical value) is applied across the junction. If an alternative voltage is superimposed over the constant one, a frequency modulation of the current through the double layer is observed. The double layer dynamics is described by a set of time-dependent Schrödinger-type equations and the self-structuring is given by means of the negative differential resistance. The results obtained from this theoretical model are in good agreement with the experimental findings.

**Acknowledgment:** This project is funded by the Ministry of Research and Innovation within Program 1 Development of the national RD system, Subprogram 1.2 Institutional Performance RDI excellence funding projects, Contract no. 34PFE/19.10.2018. Also, this work was supported by a grant of the Ministry of Research and Innovation, CNCS-UEFISCDI, project number PN-III-P4-ID-PCE-2016-0355, within PNCDI III.

**Keywords:** double layer, fireball, negative differential resistance, fractal fluid, scale relativity.

# Chaotic states of Plasma Triggered by the Nonlinear Dynamics of Simple and Multiple Double Layers

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Plasma is a complex system able to adapt to any external constraint that drive it away from equilibrium. One of the consequences is the local selfstructuring of plasma in form of fireballs or multiple double layers. When the external constraint exceeds a certain critical value, these complex space charge structures pass into dynamic state, during of which periodic disruptions of the double layers determine periodic releasing of bunches of electrons and positive ions into plasma, triggering low-frequency instabilities or, in certain experimental conditions, chaotic sates of plasma. Here, experimental results are presented showing the development of different scenarios of transitions to chaos in plasma (by intermittency, by sub-harmonic bifurcations). In certain experimental conditions, a competition of three scenarios of transition to chaos (intermittency, quasiperiodicity and cascade of sub-harmonic bifurcations) was observes in a plasma system in which a fireball exists in dynamic state. A theoretical model was developed in the frame of scale relativity theory, able to describe the observed phenomena. The equivalence between the formalism of the fractal hydrodynamics and the one of Schrödinger-type equation is established. The different criteria of evolution to chaos are obtained by applying the full and fractional revivals formalism.

**Acknowledgment:** This project is funded by the Ministry of Research and Innovation within Program 1 Development of the national RD system,

Subprogram 1.2 Institutional Performance RDI excellence funding projects, Contract no. 34PFE/19.10.2018. Also, this work was supported by a grant of the Ministry of Research and Innovation, CNCS-UEFISCDI, project number PN-III-P4-ID-PCE-2016-0355, within PNCDI III.

Keywords: fireball, multiple double layer, fractal hydrodynamics, scale relativity.

### Supersonic crowdions and voidions

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Rapid development of new technologies is often associated with realization of non equilibrium states of materials where new mechanisms of structure evolution different from conventional ones can emerge. One of examples of this phenomenon is formation of crowdions - a particular type of an interstitial defect embedded in a close packed atomic row. It was demonstrated that crowdions can vary by configuration and velocity being able to promote in supersonic and subsonic mode. Compression in part of the crystal naturally initiates the rarefaction in the neighboring zone arousing formation of vacancies and their delocalized variations - voidions able to travel along closely packed crystallographic directions.

The paper presents a state of the art overview on recent advances in the field of crowdion and voidion dynamics in crystals as investigated by means of atomistic simulations. Presented results can be useful for providing new insight in the analysis of defect rearrangement and accumulation in severely impacted materials.

KEA thank for the financial support the Council of the President of the Russian Federation for state support of young Russian scientists, grant No MD-3639.2019.2 IASh acknowledges the financial support of Russian science foundation, grant No 19-72-00109

**Keywords:** atomistic simulations, nonlinear dynamics, crystal lattice, crowdion, voidion, extreme impacts.

# Dynamics of Discrete Breathers in Normal Modes in a Symmetric Lattice

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A nonlinear lattice which has a particular symmetry supports smooth mobility of discrete breathers (DBs) or intrinsic localized modes (ILMs). This lattice, that is called the pairwise interaction symmetric lattice (PISL), is an extension of Fermi-Pasta-Ulam (FPU)  $\square$  lattice and is invariant with respects to a certain map in the complex normal mode coordinate. In this study, we numerically investigate interaction between DB and normal modes in both FPU- $\square$ lattice and PISL. Difference of dynamics of DB is discussed from the viewpoint of the symmetry of lattices.

**Keywords:** Discrete breather, Intrinsic localized mode, Fermi-Pasta-Ulam  $\beta$  lattice, Pairwise interaction symmetric lattice.

### Micro-Ramp Wake Structures Identified by Liutex

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Several vortex identification methods such as  $\Delta$ , Q,  $\lambda_2$ ,  $\lambda_{ci}$ ,  $\Omega$  criteria have been widely used to capture coherent structures in turbulent flows. Recently, a new vector named Liutex was proposed by Liu's group to describe the local fluid rotation accurately. In this paper, Liutex vector is utilized in investigating the wake structures generated by a micro ramp in supersonic turbulent flow, as well as compared with other criteria which only have scalar forms. Some advantages can be observed in our application: 1. Liutex can provide both the precise swirling strength and the local rotational axis, while many vortex identification methods fail to specify the precise swirling strength and cannot identify the direction of the rotational axis. 2. Liutex can separate the rotational vortices from the shear layers, discontinuity structures and other non-physical vortex structures, especially for the flows within shock waves. 3. Based on the definition of the Liutex, the vortex core of the flow structure can be well-determined. Furthermore, on the one hand, the integrals of Liutex

components are employed to quantify the strength of the vortical structures, and it shows that the K-H vortex structures with both normal and spanwise characteristics can maintain much longer and are more capable to drive the rotation motion in the process of developing downstream. On the other hand, the wave length λ of the K-H instability is obtained according to the autocorrelation of the Liutex vector field.

Keywords: Micro-Ramp, Wake Structure, Liutex.

### Generalised univariable fractal interpolation functions

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We show how to construct a generalised iterated function system whose graph is the attractor, a fractal set, of some continuous function which interpolates a given set of data.

Moreover, Rakotch contractions and vertical scaling factors as (continuous) `contraction functions' are used in order to obtain generalised fractal interpolation functions with extensive practical applications, including data fitting and approximation of functions.

A special generalised fractal interpolation function is introduced as an explicit illustrative example to show the effectiveness of the proposed method as compared to other existing methods.

In particular, fractal interpolation functions which are widely presented in the literature can be obtained as particular cases of our construction.

**Keywords:** Iterated function system (IFS); Fractal interpolation function (FIF); Rakotch contraction; vertical scaling factor.

# A class of complex systems without equilibria with parametric control

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In the recent years, the study of systems with hidden attractors has become popular due to the complex dynamics they exhibit. In particular, multistable systems with at least a hidden attractor are usually hard to analyze. Currently, several multistable systems with coexistence of self-excited and hidden attractors are already reported. However, multistable systems without equilibria and multiple chaotic attractors are rather rare. In this work we report a class of piecewise linear systems (PWL) without equilibria that presents monostability, bistability and multistability through a single parameter control. A particular case is studied and characterized by the maximum Lyapunov exponent for different values of the control parameter.

**Keywords:** Chaotic Attractor, Multistability, Bifurcation, Hidden attractors.

# Extreme solutions of the cubic complex Ginzburg-Landau equation with nonlinear gradient terms

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Dissipative solitons, and in particular dissipative solitons with extreme spikes, are a class of oscillatory solitons that have been observed and studied in the cubic-quintic complex Ginzburg-Landau equation (CGLE), both in the anomalous and normal dispersion regimes. Nevertheless, we may disregard the quintic terms of the CGLE and instead add the so-called gradient terms to obtain both stable plain dissipative solitons and oscillatory solutions with extreme spikes. The latter were observed numerically but a careful study of their characteristics was not yet performed. Here, we will investigate different generalizations of the cubic CGLE that admits stable solitons in order to find oscillatory solutions, finding regions of existence and bifurcation diagrams. Then, we will study the dynamics of those solutions in the presence of several gradient terms and also third order dispersion, exploring which terms enhance or depress this behavior.

**Keywords:** Cubic Ginzburg-Landau equation, gradient terms, dissipative solitons, solitons with extreme spikes, rogue waves.

# Design Hidden Bifurcation kind to Multiscroll Chaotic Attractors via Saturated Function Series

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In this paper, design Hidden Bifurcation kind to Multiscroll Chaotic Attractors via Saturated Function Series are reconnoitred. The idea was taken from the work Faiza, et all. (2019) and the method introduced by Menacer, et al. (2016) for Chua multiscroll attractors. These idea (hidden bifurcation kind) depends on how you appear. The scrolls odd or even were the number of scrolls is even. We have studied many examples to "n" of scrolls satisfies 'n=p+q+2' in LÜ, Chen et all (2004).

**Keywords:** Hidden bifurcation, multiscroll chaotic attractor, saturated function series.

# Analysis of thermal and quantum escape times of Josephson junctions for signal detection

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Josephson junctions are interesting as detectors, for they can reach the quantum sensitive limit [1,2]. Moreover, as they are superconducting elements, the temperature can be lowered as much as cryogenics allow to minimize thermal noise. On this basis, it is maintained that Josephson junctions hold promises for the detection of very weak electromagnetic signals, possibly close to the single photon limit. However one encounters a number of difficulties connected to the quantum nature of the superconducting phase and the nonlinear confining potential. In fact the

quantum phase of the Josephson junction cannot be directly measured, and therefore the experimentally accessible information is only connected to the appearance of a voltage when a special threshold is reached. While thermal escapes can be kept at bay lowering the operating temperature, the contribution to the escapes through tunnel remains at any temperature and eventually becomes statistically dominant. In this work we show how the limits to detection can be embedded in the frame of signal detection [3]. As a consequence, the optimization of the detection probability (and the minimization of the false alarm probability) gives a guide to select the Josephson junction parameters that best suit to reveal weak microwave signals fields.

**Keywords:** Josephson Junctions, Escape time, Signal Detection. **References:** 

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# Control for Set-valued Movements of Dynamical Systems under Uncertainty with Applications

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The problems of guaranteed control for nonlinear dynamical systems with uncertainty in initial states and parameters are studied. The case is investigate when only the bounding sets for initial system states and for system parameters are given without any additional statistical or probabilistic information on these values. Applying the previously

developed approaches to evaluating trajectory tubes and reachable sets, we establish the properties of optimal control that solves the problem of control for the trajectory tube of a dynamic system with uncertainty and nonlinearity of a quadratic type.

**Keywords:** Nonlinear dynamics, Control, Estimation, Uncertainty, Ellipsoidal calculus, Funnel equations.

# The Wonderful World of Flatbands: From Basics of Compact Localized States to Caging of Classical and Quantum Interactions

#### Sergej Flach

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Certain lattice wave systems in translationally invariant settings have one or more spectral bands that are strictly flat or independent of momentum in the tight binding approximation, arising from either internal symmetries or fine-tuned coupling [1]. These flat bands support compact localized eigenstates (CLS) and display remarkable strongly interacting phases of matter. Originally considered as a theoretical convenience useful for obtaining exact analytical solutions of ferromagnetism, flat bands have now been observed in a variety of settings, ranging from electronic systems to ultracold atomic gases and photonic devices [1],[2]. A finetuning of additional nonlinear interactions allows to continue CLS into compact discrete breathers [3]. Combining finetuning for lattices with All Bands Flat and classical and quantum interactions results caging of interactions [4], and ultimately in the explicit derivation of Many-Body-Flat-Band Hamiltonians [5].

**Keywords:** Flatbands, compact localized states, caging, fine tuning, classical and quantum many body interactions, compact discrete breathers.

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#### Battle of Salamis: Greeks were destined to win

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In this research we present a new nonlinear discrete dynamical system trying to model the battle of Salamis (480 BC) in the sense of Arms Race. The model describes the most effective strategic behavior between two opponents during a battle. Moreover, we compare the results of the Dynamical System Analysis to Game Theory, considering this conflict as a dynamic game. Specifically, the model approaches short – term conflicts between two participants (players), where the one is weaker than the other opponent. We link this attempt to the "Hawk - Dove" game and we compare the Nash equilibriums to the fixed points of the model. The purpose of this research is to investigate and to prove, mathematically, that Greeks were destined to win! The historical events of the naval battle are presented, and some hypothetical scenarios are given (e.g. what would it be the result if the battle would have taken place in another geographical location or how many damages would have been caused to each other to turn the outcome of the conflict). Having based our analysis on Chaos theory, we studied the stability of the equilibrium points and we interpret the optimal strategic behavior of each opponent. In conclusion, the proposed model can be applied in several scientific fields and simulates the way they (strategically) behave in a competitive environment.

**Keywords:** Battle of Salamis, Discrete Dynamical Systems, Modeling Strategic Behavior, Game Theory.

### Sensitivity Analysis of volume integral of Liutex in transitional flows

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Recently, Liutex is proposed as a new physical quantity to represent the local rotational part of fluids and has been classified as one of the third generation of vortex definition and identification methods. In this paper, the volume integral of Liutex in boundary-layer transition on a flat plate is examined, since the massive generation of Liutex has been widely observed in transitional flows. The sensitivity analysis of the volume integral of Liutex to direct forcing of Tollmien-Schlichting waves is performed using discrete adjoint methods. The derivation of the exact discrete adjoint of the direct numerical simulation (DNS) code is elaborated and the instability of the adjoint in chaotic systems is eliminated by the non-intrusive least squares shadowing (NILSS) algorithm. This sensitivity analysis sheds light on the effect of Tollmien-Schlichting waves on the generation of Liutex in boundary-layer transition.

Keywords: Liutex, Sensitivity analysis, Adjoint methods, Chaotic system, Transitional flows.

Invited Workshop: Liutex and Third Generation of Vortex Defintion and Identification, Chair: Professor Chaoqun Liu.

### Atoms and pseudo-atoms in quantum measure theory **Alina Gavrilut**

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In this paper, different results concerning (pseudo)-atomicity are obtained from the Quantum Measure Theory mathematical perspective and several physical applications are given. Precisely, the mathematical concept of atomicity (and, particularly, that of minimal atomicity) is extended, based on the non-differentiability of the motion curves associated to the motions of the structural units of a complex system on a fractal manifold.

**Keywords:** Atom, Pseudo-atom, Minimal atom, Non-differentiability, Fractal manifold.

# Life evolves in experimentally confirmed 'half-chaos' of not fully random networks, but not 'on the edge of chaos'

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An experiment cannot refute mathematical theory, but decides whether a given theory is suitable for describing the indicated phenomena. I do not question the theory of chaos in the field of infinite and continuous spaces, but I present a hitherto unknown class (half-chaotic systems) of finite discrete networks in which this theory gives erroneous results and I explain why. It is easy to repeat the computer simulation experiments described here, also much wider descriptions are available. The widely known Kauffman hypothesis 'life on the edge of chaos' is based on the recognition made in random autonomous Kauffman networks. This recognition, supported by mathematical theory of chaos, gave an image in which systems can be either ordered or chaotic with a fairly fast phase transition between them. Only for parameters in the immediate vicinity of this phase transition, the changes have properties suitable for describing stability of adaptive evolution and typically modeled objects. This limitations for the variables are strong. However, modeled adapted systems are not fully random, they are usually stable, but the estimated parameters are usually "chaotic"-they place the fully random networks in the chaotic regime, far from the narrow phase transition. The half-chaotic network has such "chaotic parameters", it simultaneously exhibits in similar share both small (ordered) and large (chaotic) reactions for small disturbances. The discovery of half-chaos frees modeling of adapted systems from sharp restrictions; it allows to use "chaotic parameters" and get a nearly stable system more similar to modeled one. It gives a base

for identity criterion of an evolving object, simplifies the definition of basic Darwinian mechanism and changes "life on the edge of chaos" to "life evolves in the half-chaos of not fully random systems".

**Keywords:** Kauffman networks, complex networks, chaos, life on the edge of chaos, phase transition to chaos, damage spreading.

# Assessing observability from recorded data using Delay Differential Analysis (DDA)

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When studying a complex, high dimensional system often only a single variable may be measured. In theory all information about the original system can be obtained by using delay or differential embeddings, however in practice, some variables will have more dynamical information about the original system than others. Variables that can recover all states of the original system are said to be fully observable. While the observability of some measurement function can be determined analytically, numerical methods for assessing observability from a time series alone are desirable, as the equations for the underlying system are often unavailable. Here we extend DDA, a nonlinear time series analysis framework, to assess observability. DDA projects time series data onto a low-dimensional subspace made up of features from fitting nonlinear delay differential embeddings. We show that using the least squares error of this fit tracks observability, as compared with analytical ground truth for some well-studied nonlinear dynamical systems. This approach is promising for ranking recorded variables in high dimensional data, such as in brain data and other biomedical recordings.

**Keywords:** observability, Delay Differential Analysis, chaotic systems.

# A Statistical Ensemble Based Approach for Entropy in **Cryptocurrencies Markets**

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In this paper, we consider the cryptocurrencies market and in particular, we try to point out an "affinity" between the system of agents trading in cryptocurrencies and statistical mechanics. We focus our study on the concept of entropy in the sense of Boltzmann and we try to extend such a definition to a model in which the particles are replaced by N economic subjects (agents). The agents are completely described by their ability to buy and to sell a certain quantity of cryptocurrencies. We provide some numerical examples by applying the model to the closing prices of the main six cryptocurrencies and we show that entropy can be used as an indicator to forecast the price trend of cryptocurrencies.

Keywords: Cryptocurrency, Entropy, Prices Forecast, Boltzmann, Blockchain.

### Josephson-based Threshold Detector for Lévy-**Distributed Fluctuations**

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We present a study for the exploitation of a Josephson junction to detect, in a noisy background, the presence of Lévy distributed fluctuations [1]. In our analysis we simulate Lévy distributed fluctuations by adding a noisy current to a linearly ramped bias current. This causes clear modifications of the switching current distribution out of the zero-voltage state of the junction [2-4].

By numerically solving the resistively and capacitively shunted junction (RCSJ) model equation, which describes the dynamics of the phase difference across the junction, we find that the cumulative distribution function (CDF) of the switching currents is modified depending on both the stability index  $\alpha$  and the intensity of the Lévy noise fluctuations [5]. The analysis of the CDFs obtained for noise sources with different statistical features allows to associate to each CDF a specific value of  $\alpha$ , while providing information on the Lévy statistics of the noise sources.

This analysis provides a scheme for an experimental setup, where a Josephson junction is employed as a detector for Lévy distributed fluctuations embedded in a noisy background.

Keywords: Josephson junction, RCSJ model, noise detector, Lévy noise, switching current distributions, cumulative distribution functions.

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## Effect of Harvesting on Extinction Time in a Stochastic **Population Model**

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Deterministic population models have limited predictive power since they do not take into account the random characteristics of environmental impacts. Therefore, we use stochastic approaches for a realistic description of a population. There are two commonly used methods to consider stochastic effects. One is to focus on Fokker Planck solutions by identifying statistical properties of noise and writing a Langevin equation. Another approach is to solve the Master equation with the transition rates representing the transition of the population from n individuals to n+r individuals. Thus, it is possible to calculate Mean Time to Extinction (MTE) for single-step or multiple-step processes using WKB (Wentzel-Kramers-Brillouin) approximation. In this study, we first expand a population model under weak or strong Allee effect, according to the value of the parameters to include a harvesting function. Since the effect of harvesting function on MTE can only be analyzed based on multiple-step processes, we propose a way to use WKB approximation for multiple-step processes. Thus, we examine the effect of stochastic effects on extinction time on the basis of multiple-step processes.

**Keywords:** Stochastic Population, Allee Effect, Mean Time to Extinction, WKB approximation.

# Study on eddy current of all-oxygen reheating furnacebased on Liutex vortex recognition method

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The diffusion combustion effect can be realized by using the special burner structurein the total oxygen heating furnace, thus the combustion temperature can be uniform and the industrial heating application can be realized. The results show that the essence of dispersion combustion is to form a new mixed combustion phenomenon by flue gasreflux and co-combustion agent. There are a lot of turbulent vortex structures in the mixed flow process, and the structure characteristics of vortex directly determine the dispersion combustion effect. In this paper, the influence of vortex structure on themixing is analyzed by numerical simulation of the experimental full oxygen dispersion heating furnace. It is found that the reflux mixing process of the full oxygen dispersion combustion is mainly concentrated at the outlet of the nozzle to the furnace withthree molecules. So this paper focuses on Q guidelines and Liute x vortex identification method is used to analyze the characteristics of vortex structure formation

in the first half of furnace and its relationship with dispersion combustion. The results showthat the Q criterion is consistent with the position of the large vortex identification and the Liutex vortex identification, but it can not be shown for the small vortex structure which determines the dispersion combustion. It is found that the asymmetric nozzle used in the experiment is more easy to achieve dispersion combustion. The large vortex formed by turbulent reflux flue gas dilutes the concentrationof combustion aid, and the diffusion motion of a large number of small vortices realizes the mixing of fuel and combustion aid, so the dispersion combustion can be realized by controlling the burner structure, which provides guidance for industrial application.

Keywords: numerical simulation; Asymmetric nozzle; Vortex Structure; Liutex vortex identification method.

# **Empirical scaling and dynamical regimes for GDP:** challenges and opportunities

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We explore the distribution of gross domestic product (GDP) and per capita GDP among different countries in order to elucidate differences in the dynamics of their economies. An initial analysis of GDP data and per capita GDP data from 1980 and 2016 (and many years in between) typically finds three scaling regions - signatures of likely differences in dynamics. The GDP of the largest ~25 economies (nations, EU) follows a power law GDP ~ 1/rank (c.f. Garlaschelli et al. 2007); followed by a second scaling region in which GDP falls off exponentially with rank and finally a third scaling region in which the GDP falls off exponentially with the square of rank. This broad pattern holds despite significant changes in technology (enormous growth in computing power, "intelligent" automation, the Internet), the size of the world economy, emergence of new economic powers such as China, and world trade (almost free communication, containerized shipping yielding sharp declines in shipping costs, trade partnerships, growth of the EU, multinationals displacing the traditional economic role of nation-states. Thus, empirically, these patterns may be universal in which case one approach to growth of less developed economies (in the second and third scaling regions of per capita GDP) may be to identify and target causative differences between these economies and those in the first (power law) scaling region. For example, Montroll and Shlesinger (1982) suggest a basic lognormal distribution as a consequence of multiplying many independent random variables, with a power law high-end tail because "the very wealthy generally achieve their superwealth through amplification processes that are not available to most." On the other hand, Reed and Hughes (2002) show how power law behavior can arise if "stochastic processes with exponential growth in expectation are killed (or observed) randomly."

**Keywords:** Economic models, Scaling, Scaling regions, Power law, Economic growth, Inequality.

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# Investigation of Flow Structures around Cylinders with High Reynolds Number by Liutex Vortex Identification Methods

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According to Liu et al, the development of vortex identification method can be classified into three generations. The first-generation methods are based on the vorticity. The second generation of vortex identification methods, including Q,  $\lambda 2$ ,  $\Delta$  and  $\lambda ci$  criteria. In 2017 and 2018, a Liutex vector (previously named Rortex) was proposed to provide a mathematical and systematical definition of the local rigid rotation part of the fluid motion, including both the local rotational axis and the rotational

strength. As a new physical quantity raised by Liu et al, Liutex has direction and magnitude. Its direction represents the local vortex line and is parallel to the normal vector of the vortex iso-surface. Its magnitude is exactly the local angular speed, and Liutex represents a force which is the driving force of turbulence generation. In this paper, the computational results of flow around cylinders using PANS method based on Menter shear stress transport (SST) turbulence model were presented in this investigation. Partially averaged Navier-Stokes (PANS) method derived from the traditional Reynolds averaged Navier-Stokes (RANS) methods by introducing controlling parameter into original RANS equations to modify these equations, is one kind of hybrid methods which can effectively simulate the separated turbulent flows. The Liutex was used to identify vortex structures in late boundary layer transition on low around cylinders which is simulated by a SST-PANS code. Several comparative studies on simple examples and realistic flows are studied to confirm the superiority of Liutex.

**Keywords:** Partially averaged Navier-Stokes (PANS), Reynolds Number, Flow around Cylinders, Liutex vortex identification methods.

# Singular amplification of low-frequency fluctuations in optical spectra of 4He quantum liquid

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The optical spectra of impurity atoms (molecules) in 4He quantum liquid are considered. It is shown that the main distinguishing property of the liquid phase to maintain local pressure in all its macro- and meso-regions leads to a huge increase in the contribution of low-frequency density fluctuations to the electronic transitions of impurity atoms, diverging as with a decrease of the frequency of fluctuations. As a result of this divergence, the zero phonon line (ZPL) in the optical spectrum of an impurity atom acquires finite broadening and asymmetric shape already in the limit of zero temperature. Another consequence of this divergence is an abnormally strong dependence on the width and shape of ZPL on the strength of the vibronic interaction of the electronic transition. The optical

spectra observed in [1, 2] for impurity atoms of Au, Cu, Cs, and Dy in liquid 4He at low temperatures are explained.

**Keywords:** Quantum liquids, density fluctuations, zero-phonon lines. **References:** 

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# Numerical Simulation of Leakage Flow inside Shroud and Its Interaction with Main Flow in an Axial Turbine

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In a turbine, the leakage loss is an important source of turbine aerodynamic loss. In this paper, steady numerical simulations were performed to analyze the flow characteristics in the shroud of a 1.5-stage axial turbine and the interaction between the leakage flow and the mainstream. A spatial loss audit inside shroud was undertaken and the loss due to the interaction was assessed. Based on the Liutex method, the vortices in the shroud were identified. The results show that the leakage flow in the shroud could be regarded as a jet process with several parts which are free jet and wall attached jet. The rest part of the shroud was mostly occupied by the vortices. There is both inflow and outflow on the interface of outlet cavity and mainstream, accompanied by a series of vortices in the outlet cavity. The tip leakage loss was mainly caused by the mixing process between the leakage flow and the mainstream in the outlet cavity and downstream. The difference in tangential velocity of them created a significant part while the vortices in the shroud produced little loss.

**Keywords:** Flow characteristics, Vortex, Turbine, Shourd, Leakage loss, Liutex method.

# Application of Liutex for Analysis of Complex Wake Flow Characteristics of Wind Turbine

#### Yang Huang, Decheng Wan

State Key Laboratory of Ocean Engineering, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai, China Wind energy is believed to be one of the most promising renewable energy to substitute the conventional fossil fuel. In order to improve the power generation capacity of the wind farm, a common strategy is to increase the number of the wind turbine in the selected wind farm. However, the annual energy output of the wind farm don't linearly increase with the total number of wind turbines clustered in the wind farm. The main reason is that the wake loss of wind turbine may be enlarged with the decrease of inter-turbine spacing. The downstream wind farm located in the wake of the upstream wind turbine experience decreased wind velocity and increased turbulence inflow, resulting in the decrease of power output and increase of fatigue loads. One important principle of the wind farm layout is to minimize the effects of wake loss. It is necessary to investigate the wake characteristics of wind turbine in detail. Due to the rotating movement of the blades, there are increased turbulence level in the wake field of wind turbine. Affected by the wake flow, the aerodynamics of downstream wind turbine are significantly altered, which furtherly increases the instability of the flow field. In order to better understand the vortex generated by the rotating blades and the development of wake flow, the third generation of vortex identification method Liutex is utilized to capture the vortex structure and illustrate the flow characteristics in the present work. Detailed vortex characteristics including the rotational axis, vortex core center location and vortex core size be well explained by this new third generation of vortex identification method. In this paper, the actuator line technique is applied for the aerodynamic simulations of the wind turbine. The Liutex vector is calculated and used to detect the wake characteristics of wind turbine. Different wind turbine types including the onshore wind turbine and the floating wind turbine are taken into consideration. In addition, the vortex structure represented by Q is compared with that calculated from Liutex. The influence of wake vortex on the turbulence intensity and aerodynamic loads is also investigated. Some conclusions can be drawn from the analysis results, which help to illustrate the complex wake characteristics of wind turbine.

**Keywords:** Wind tubine, Wake flow, Vortex structure, Actuator line model, Turbulence, Liutex vector.

Vibrational Resonance in inhomogeneous and spacedependent nonlinear damped systems

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The properties of nonlinear systems have attracted a considerable interest these past years since they can be used to develop bio-inspired applications ranging from signal detection to image processing. Among these nonlinear signatures is Vibrational Resonance (V.R.) which expresses the ability of a nonlinear system to take benefit of a high frequency perturbation in order to enhance its response to a weak low frequency excitation [1,2]. Since its introduction in mechanical systems [1] and electronic devices [2], this effect has been widely reported in various systems and for different applications [3]. Indeed, V.R can be used to detect subthreshold signals [4] or to enhance the perception of subthreshold images [5]. Among the systems which exhibits V.R., we can cite bio-inspired circuits [6,7], networks of coupled electronic neurons [8], as well as inhomogeneous and spacedependent nonlinear damped systems [9]. These last mechanical systems constitute the subject of our communication which adresses more specifically the conditions of existence of V.R. The system under numerical investigation models a particle of mass m submitted to a multistable potential and a space dependent nonlinear damping. After showing that multiple resonances can be controlled by the system parameters, we show that the particle mass can allow or hinder V.R. Indeed, V.R. ceases to exist for particles exceeding a critical mass [10].

**Keywords:** Vibrational Resonance, nonlinear dynamics.

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# Dynamics of the charge transfer through a memristor between two initially charged cells

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A charge-controlled memristor model, with the memristive expression M(q) defined as the ratio between the voltage through the memristor and the current flowing into it, is used in the quantitative study of the charge transfer between two cells initially charged with known voltages. Calling q0 the total charge having flown into the memristor since its first use – illustrating the memory feature of this component-, we show that the derivative of M(q) versus q must be a continuous function of q, leading us to improve the expression for M(q). Then, the dynamics of this charge transfer, as well as the time evolution of the voltages of both cells, is analyzed analytically and confirmed by numerical experiments.

# Energy Efficiency of Cortical Action Potential Generation at Different Temperatures

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The brain is a complex system that generates human intelligence, whose information processing is remarkably energy consuming. The human brain accounts for merely 2% of body weight, yet consumes about 20% of metabolic energy. The issue of energy efficiency is therefore vital and worth studying, as abnormal energy metabolism of the brain will seriously impair physical health. Here we use the method of numerical simulation to quantitatively describe the energy consumption of the nervous system, especially that of cortical action potential generation. We add impacts of Gaussian white noise and external stimulation current to the Hodgkin-Huxley-Style cortical neuronal model in order to simulate the noisy internal environment and synaptic transmission, derive formulas for measuring energy efficiency, and finally discuss three indicators including Information Rate(IR), Energy Consumption(EC) and Energy Efficiency(EE) affected by different factors. Our results indicate that the optimized temperatures at which cortical neurons' IR and EE reach a maximum both occur from 36.5 to 36.7oC. It coincides with the normal temperature of the human brain, thus giving the optimization of the brain mechanism during human evolution a strong theoretical support.

**Keywords:** cortical neuronal model; energy efficiency; entropy of information; energy consumption.

# Fractal basin boundaries on to chaos in a Josephson junction model

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The Josephson junction model provides a playground to test the nonlinear aspects of the familiar pendulum. We are concerned about the coexistence of the fractal basin boundary, and chaos in the phase space of the Josephson junction model (JJ) as a nonlinear oscillator. This was first studied both theoretically, and experimentally by lansiti et al. [1]. Onset of chaos in JJ has also been studied [2]. Synchronizing with an external radiation is an idea present in what is known as a Shapiro step.

The presence of the fractal basin boundary makes it difficult to predict the attracting step. We can characterize the phase space by studying the synchronizing properties of oscillators from the fractal basin, and the chaotic region. The two oscillators can have a drive-response role relative to each other, as Pecora and Carroll introduced [3]. We investigate the nature of the phase space from this synchronizing stance. References:

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# Appearance and Disappearance of 2-D Spatiotemporal Extreme Waves in Quadratic Nonlinear Medium

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Solitons are special nonlinear waves studied since the 1844s, in several physical domains, and characterized by their propagation in absence of distortion in time, space and spectrum. In optics, other types of extreme waves have been studied so far in cubic and, more recently, in quadratic nonlinear media, with the particularity to appear and disappear without trace. Akhmediev breathers, Peregrine solitons are particular solutions of the Nonlinear Schrödinger equation able to also explain in a deterministic way the observation of rogue waves but, so far, they are reported in optics mainly in one-dimensional system. In this paper, we show how spontaneous extreme waves may be experimentally observed in a crystal with quadratic nonlinearity involving two transverse dimensions. These extreme waves are excited by using wide input optical beams propagating in type-II quadratic KTP crystal in presence of large spatial walk-off. When increasing gradually the input power, above the well-known regime of 2D spatial trapping, we observe first the appearance of these extreme waves

and then their nearly complete disappearance. We also characterized other satellite nonlinear phenomena, such as spatial ultrafast switching or cascaded periodic generation. A reduction of the temporal envelope has also been observed and used to implement an all-optical ultrafast pulse reshaping. Moreover, we demonstrate the possibility to induce periodically spaced extreme waves by using a spatially modulated input beam.

**Keywords:** Spatial soliton, Quadratic soliton, Extreme waves, Transient process.

# Anomalous scaling in the kinematic magnetohydrodynamic turbulence under the influence of helicity in the two-loop approximation

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In the framework of the field-theoretic renormalization-group approach and the operator product expansion, the influence of the helicity (spatial parity violation) on the anomalous dimensions of the leading composite operators, which drive the anomalous scaling of correlations functions of the magnetic field deep inside the inertial range, is investigated in the kinematic magnetohydrodynamic turbulence, i.e, in the model in which the Lorentz term is omitted in the stochastic Navier-Stokes equation and the magnetic field behaves as a passive vector quantity. It is shown that there is a significant qualitative difference between the role of the helicity in the problem of the passive magnetic (vector) field and the analogous problem of a passive scalar advection by the Navier-Stokes turbulence, namely, while in the scalar case the anomalous dimensions of the corresponding leading composite operators are independent of the helicity, they significantly depend of the amount of the helicity in the case of the magnetic field and, as a result, the scaling properties of the correlation functions of the magnetic field also feel the presence of the spatial parity in conductive turbulent environments.

**Keywords:** renormalization group, operator product expansion, turbulence, Navier-Stokes, stochastic, dynamics, scaling, anomalous, dimensions.

### Interaction of Limit Cycles for the FitzHugh-Nagumo Model

#### Shunji Kawamoto

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The interaction of discrete limit cycles related to the FitzHugh-Nagumo (FHN) model is discussed in this paper, from the standpoint of timedependent chaos functions. Firstly, a two-dimensional (2-D) solvable chaos map corresponding to the FHN model is derived on the basis of chaos functions, and 2-D chaotic maps with one system parameter are presented for generating limit cycles. Secondly, two limit cycles are set on the xn-yn plane as nonlinear dynamics of two neural cells, and the propagations are illustrated with a constant velocity in opposite direction. Finally, the interaction of two limit cycles is numerically considered by adding interaction terms in a 2-D system A&B composed of the 2-D chaotic maps A and B. Then, the 2-D system A&B with two stable limit cycles is shown to have chaotic dynamics depending on the interaction terms, as a complex system of non-equilibrium open systems.

**Keywords:** The Van der Pol oscillator, The FitzHugh-Nagumo model, Chaos function, Limit cycle, Interaction, Complex system, Non-equilibrium open systems.

## Limit Cycle Analysis for 2-D Time-Dependent Logistic Maps

#### Shunji Kawamoto

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A limit cycle analysis is presented in this paper as a computer-aided approach, from the standpoint of chaos functions for two-dimensional (2-D) time-dependent logistic maps. Firstly, 2-D logistic maps are derived by extending the 1-D time-dependent logistic map based on time-dependent chaos functions, for population growth in time. Secondly, bifurcation diagrams and fixed points with one system parameter are calculated, and discrete limit cycles are numerically obtained, which are known to possess

chaotic properties of entrainment and synchronization. Finally, approximate equations to the stable limit cycle are found under approximations on recursion relations of the 2-D logistic map, and are discussed for an isolated trajectory generated by the interaction of populations, as one of non-equilibrium open systems.

**Keywords:** Logistic map, 2-D logistic map, Chaos function, Bifurcation diagram, Fixed point, Limit cycle, Entrainment, Synchronization, Nonequilibrium open system.

#### **Control of Cardiac Alternans**

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Among the pathological heart rhythms the alternans of the cardiac action potential duration (APD) is a phenomenon whose appearance is closely linked to ventricular fibrillation. The correlation of these two phenomena motivated our work which consists in carrying out the suppression of alternans by a control method. In addition, by using predictive control of the periodic dynamics, we prove that it is possible to delay the onset of bifurcations and to orient a trajectory towards a desired stationary target state. We give examples of numerical results showing the stabilization of the instable normal rhythm.

**Keywords:** Dynamical system, Cardiac alternans, Action Potential Duration (APD), Control, Bassin of attraction, Periodic orbit.

# Interregional network competition and dynamical systems

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In recent years, regional authorities have gained in autonomy and responsibility and have been increasingly making local decisions affecting the economy or the local transportation system. They use many instruments, notably local taxes and tolls. The object of this paper is to analyze some characteristics and possible implications of tax/toll competition between regions or inside regions. Many researchers have studied tax competition; for instance, Mintz and Tulkens (1985), Bayindir-Upmann and Ziad (2006) and many others. Some papers have been devoted to the study of commodity tax competition models: Kanbur and Keen (1993) and Lucas (2004) for instance. Balijepalli and Shepherd (2016), Kaufmann and Arnold (2018), Ly, T. Taxes (2019) have addressed the questions of inter- and intra- urban competition. In this work we are mainly interested in analyzing a simplified case which can be solved by semi-analytical tools. The special case expands on Khoshyaran (2002-2004). Two competing regions are considered, taxing a single good and tolling transportation. The model takes into account the distribution of the population with respect to transportation costs to the inter-regional border. Each region aims to maximize its revenue. Reaction curves can be parameterized by the difference between tolls respectively taxes. The reaction functions are piecewise continuous, multi-valued and highly nonlinear, because of the inhomogeneous population distribution. In tax/toll competition, reaction functions are usually assumed linear, leading to simple behavior and equilibria. But constraints can make them piecewise linear and induce complex dynamics, Bischi and Lamantia (2012). In the present case, the extreme nonlinearity of reaction functions induces nonlinear dynamics in the response of the agents and may exclude the existence of equilibria, and if equilibria do exist, they are unstable. Further research will aim to build a network model, based on Lebacque and Khoshyaran (2018a) and (2018b), which takes into account consumption of a single generic good, demand functions at population centers, impact on transportation of activity, distribution, assignment on the transportation network with non-constant arc costs, and node supply constraints. The model distinguishes local flows from flows induced by economic activity. Major agents (regions, cities) have the capacity to make tax/toll decisions. The above model is bi-level: major agents (regions, cities) compete in order to increase their revenue, while individuals compete for the network resources. The study of equilibria and dynamics

in this context will generalized the study carried out in the two-region simplified case.

Keywords: Nonlinear maps, reaction functions, chaotic behavior, transportation, economy, complex dynamics competition, network eauilibrium.

## Localized modes induced by distributed impurities in resonant circuit arrays

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A localized mode is generated by a distributed impurity induced by en external coil placed on a resonant circuit array. The localized mode is possibly applicable to wireless power transfer since the magnetic flux is spontaneously localized around the external coil. However, the frequency of the localized mode fluctuates for the position of the external coil. We investigated how to reduce the frequency fluctuation by adjusting the interval of resonant circuits and the diameter of the external coil. A specific design that makes the frequency fluctuation almost zero has found numerically. The circular coils in the resonant circuit array are overlapped of almost 40% of its diameter, and the diameter of the external coil is almost double that of the coil in the resonant circuit array. Also, the design is confirmed by analytical solutions which are derived for approximated models.

Keywords: Localized mode, Resonant ciruict array, Distributed impurity, Wireless power transfer.

## Stochastic properties of two-wheeled robot on soft surfaces

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Dynamics of an inverted pendulum on a wheel is a popular model of nonlinear unstable processes, which can be stabilized by a feedback control. Such control is defined by observed data using some additional sensors. Errors of measurements are natural properties of sensors. Therefore stochastic behaviors appear into the control. A mathematical model for a two wheeled robot on soft surfaces contains a differential inclusion and process of the control with stochastic properties. We study this model by analytic and numerical approaches under a feedback control. As a result we show stabilization of the robotic equipment on a hysteresis loop and estimate a mean time for staying of the system near an unstable point.

Keywords: Stochastic process, differential inclusion, robotics, nonlinear dynamics.

### On solvability of the equation $\lambda f - w(f \circ \phi) = g$

### **Arkady Kitover**

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Let K be a compact Hausdorff space, C(K) be the space of all continuous complex-valued functions on K,  $\phi$  be a homeomorphism of K onto itself, and w∈C(K). We provide necessary and sufficient conditions for the equation  $\lambda f - w(f \circ \phi) = g$  to have a solution  $f \in C(K)$  for any  $g \in C(K)$ . As a consequence we provide solution for similar problems in spaces of measurable functions.

## Analysis of discrete breathers in the mass-in-mass Chain in the state of acoustic vacuum

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Present study concerns the dynamics of special localized solutions emerging in the mass-in-mass anharmonic oscillatory chain in the state of acoustic vacuum. Each outer element of the chain incorporates an additional, purely nonlinear mass attachment. Analytical study of the later, revealed the distinct types of stationary discrete breather solutions. Along with the analytical description of their spatial wave profiles we also establish their zones of existence in the space of system parameters. Stability properties of these solutions are assessed through the linear analysis (Floquet). All analytical models are supported by the numerical simulations of the full model.

**Keywords:** Acoustic vacuum, Mass-in-mass chain, Breather solutions. This work was supported by Russian Foundation for Basic Research according to the research project no. 18-03-00716.

## On the asymptotics of the solution of a Klein-Gordon-Fock equation with a variable coefficient for the Laplacian

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We study the problem of wave propagation in the medium whose velocity characteristics change under an external impact. We will study a three-dimensional case. The aim of our study is constructing the asymptotic of the solution for the Klein-Gordon-Fock operator with a variable coefficient of the Laplacian at infinity. The case of holomorphic coefficients is considered.

**Keywords:** asymptotic, holomorphic coefficient, Laplace–Borel transform, Weighted Spaces.

Localization of negative-effective-mass electron by supersonic kink in 1D lattice

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Supersonic kink in nonlinear atomic chains with realistic interatomic potential produces local compression of the lattice. Lattice compression enhances electron Fermi energy and produces for an electron a local potential hill through the deformation potential of the proper sign. Such potential hill can localize (trap) negative-effective-mass (NEM) electron and such unconventional trapping can be observed in the simplest tightbinding model of electron band [1]. Here we confirm numerically the possibility of the trapping of NEM electron with the energy above the top of its tight-binding band by supersonic kink in the nonlinear lattices with two different interatomic potentials, the  $\alpha$ - $\beta$  FPU and Morse, and with the proper electron-phonon interaction. We reveal that the localization length of the electron wave function is much larger than lattice period in the case of adiabatic electron dynamics, electron localization length saturates for small-amplitude kink and continuously decreases with the increase of the velocity of the ultradiscrete supersonic kink. The ultradiscrete supersonic kinks, which were revealed in the nonlinear lattices with different interatomic potentials with hardening anharmonicity, have approximately sinusoidal envelope with the "magic" wave number [2,3]. The compression supersonic kink in the lattice with realistic asymmetric interatomic potential is accompanied by the local lattice expansion [2], which can trap positiveeffective-mass electron. The work of Yu.A.K. was funded by the Russian Foundation for Basic Research according to the research project No. 18-29-19135 mk.

**Keywords:** Supersonic kink, electron, deformation potential, localization length.

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### **Coupled State Formation in Plasma Waves**

## Ioannis Kourakis<sup>1</sup>, Ibrahem Elkamash<sup>2</sup>, Michael McKerr<sup>3</sup>, Theodoros Horikis<sup>4</sup> and Dimitri J Frantzeskakis<sup>5</sup>

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Localized patterns emerging from electrostatic wave-wave interactions in a two-dimensional plasma-fluid dynamical model are considered. The model is reduced to a Kadomtsev–Petviashvili-II (KP2) equation, a model equation arising in the description of shallow water waves. The ab initio relevance with interacting localized structures (crossing states) earlier predicted in nonlinear optical patterns is investigated. Different types of X-or Y- type coupled-wave patterns (line solitons) are shown to exist. This study reveals the similarity between large-amplitude electrostatic pulse formation in plasmas and pattern formation in nonlinear optics and in hydrodynamics, thus laying the first building blocks for a theory of coupled electrostatic states in plasmas as precursors of freak wave formation.

Acknowledgment: Support from Abu Dhabi Department of Education

**Acknowledgment:** Support from Abu Dhabi Department of Education and Knowledge (ADEK) in the form of an ADEK Award for Research Excellence 2018 (AARE-2018) grant is gratefully acknowledged.

**Keywords:** Rogue waves, freak waves, Kadomtsev–Petviashvili equation, electrostatic waves in plasmas, solitons.

### **Electrostatic Rogue Waves in Plasmas**

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We have undertaken a theoretical study from first principles of the conditions for the occurrence of rogue (freak) waves (RWs) associated with electrostatic modes in plasmas. As a case study of interest, and actually motivated by recent experiments, we have considered a multifluid model for a plasma consisting of electrons and positive ions in addition to a secondary (negative) ion population, of arbitrary mass and charge. For simplicity, the ionic temperature is neglected, and so is the

electron inertia. Relying on a multiscale perturbative technique, an evolution equation is obtained for the envelope (amplitude) of the electrostatic potential. The existence profile for RWs is investigated in terms of relevant plasma configuration parameters, namely the ion-to-ion density and mass ratio(s). It is shown that extreme excitations in the form of either the Peregrine soliton form or Akhmediev breathers may exist in a wide range of intrinsic plasma parameter values. The outcomes of this study match and in fact extend previous results based on experimental observations, are confirmed by numerical simulations which show the spontaneous formation of Peregrine soliton-like waveforms in the plasma. Our results lay the general foundations, beyond strictly speaking negativeion plasmas, for the generic modeling of freak waves in plasmas - and in fact show that such extreme events are possible in a wide range of parameter values, beyond the conditions so far explored experimentally. Acknowledgment: Support from Abu Dhabi Department of Education and Knowledge (ADEK) in the form of an ADEK Award for Research Excellence 2018 (AARE-2018) grant is gratefully acknowledged.

Keywords: Rogue waves, freak waves, Peregrine soliton, Akhmediev breather, electrostatic waves in plasmas, nonlinear Schrodinger equation, envelope solitons

## Oscillating System under Limited Excitation from **Generator or Wave Field**

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The purpose of our work is study an oscillating system and an electrodynamical transducer, which are driven either by the amplifier or wave field. In the first case an amplifier is considered as a self-exciting system with a limited power. Electrical current produced by it is converted by the transducer into mechanical force, which leads to vibrations of the base. A mechanical oscillator is mounted on the transducer base. The influence of oscillator vibrations on the formation of the driving force leads to a number of specific effects, in particular, to the Sommerfeld -Kononenko's effect. New nonlinear effects in the coupled shaker-oscillator system are studied in details. Steady-state regimes of the constructed model are investigated by methods of the theory of dynamical systems. Expressions for supplied and consumed powers are shown and investigated for regular and chaotic regimes. The invers! e problem model is also discussed. The classical results for wave power absorption by wave energy extractor as a single degree of freedom system are presented in the second considered problem. The example includes an axisymmetric buoy which oscillates and is subjected to its natural hydrostatic restoring force. Main attention is focuses on the values and expressions for the mean powers. The expression for the maximum mean power is given for the considering system.

**Keywords:** Sommerfeld – Kononenko effect, steady –state regimes, energy extractor.

Chaos in Nonideal Dynamic Systems, Prof. A. Yu. Shvets.

## Cardiorespiratory System as Nonideal System with Limited Excitation

## Tatyana Krasnopolskaya, Evgeniy D. Pechuk, Mariia O. Rudnytska

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It is known that the functioning of the heart is closely related to the dynamics of the respiratory system, and modeling their interaction is an important and relevant task. The heart of a healthy person works irregularly. Based on the DeBoer model, the interaction between the cardiovascular and respiratory subsystems was considered with regard to the equations of direct (from respiratory to cardio) and feedback (from cardio to respiratory) relations of the subsystems. So that, the cardio subsystem is under limited excitation from the respiratory subsystem. The respiratory system was modeled as a Zaslavsky generator, which is described by the known equations of the Zaslavsky dissipative standard map. Chaotic modes were revealed, which were produced by the interaction between the subsystems. It was proved that the irregularity of the behavior of phase trajectories depends on the intensity of the effect! produced by the heart rate on breathing, which is characteristic of the dynamics of the cardiovascular system of a healthy person.

**Keywords:** Heart rate, Cardiovascular system, Respiratory system, Feedback, Chaos.

# A model of stabilization of chaotic wave processes in complex dynamical systems from the point of view of the matrix decomposition theory

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A general model of the origin and evolution of chaotic wave processes in complex systems based on the proposed method of matrix decomposition of operators of nonlinear systems is developed in the article. The proposed model shows that the effect of self-organization in complex systems of different physical nature (for example, hydrodynamic, electronic and physiological ones) is based on the interaction of nonlinear processes of higher orders leading to stabilization (to the finite value) of the amplitude of chaotic wave process. Mathematically, this means the synchronous "counteraction" of nonlinear processes of even and odd orders in a general vector-matrix model of a complex system being in a chaotic mode. The implementation of the vector-matrix decomposition by means of computational experiments shows that the model of L.D. Landau describes the scenario of the occurrence of chaotic modes in complex systems quite well. It is noted that the regime of hard self-excitation of nonlinear oscillations in complex systems leads to the appearance of a chaotic attractor in the state-space. Moreover, the proposed vector-matrix model permits to find more general conditions for the origin and evolution of chaotic wave processes and, as a result, to explain the appearance of coherent nonlinear phenomena in complex systems.

**Keywords:** complex nonlinear dynamical system, state-space, chaotic attractor, matrix series in state-space, general vector-matrix model of chaotic wave processes, mode of hard self-excitation of nonlinear oscillations, stabilization of the amplitude of chaotic process.

Spatiotemporal nonlinear phenomena in multimode optical fibers

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Complex nonlinear dynamics of multimode systems is a research field that has emerged over the last years, and rapidly attracted a great research attention. In this context, beam propagation in graded-index multimode optical fibers (GRIN MMFs) are currently enjoying a strong resurgence of interest [1]. Indeed, they offer an ideal and natural environment for investigating collective dynamics based on complex interactions among a large number of basic elements. From the application viewpoint, MMFs may provide instead interesting solutions, beyond the current capabilities of single-mode fiber for power delivery or transmission capacity. In this talk, we will overview some of our recent results concerning multimode phenomena in GRIN MMFs [2], including, for instance, Kerr-induced spatial beam self-cleaning [3], geometric parametric instability [4], as well as their temporal and polarization dynamics. We will also demonstrate our very recent observation of multimode modulation instability, multimode solitons and their associated ultra-broadband dispersive waves, spontaneously generated from the pulse breaking of a picosecond laser pump in anomalous dispersion regime.

**Keywords:** Nonlinear optics, multimode optical fibers, spatiotemporal phenomena.

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## Application of ultrawideband chaotic signals for wireless ranging

### Lev Kuzmin, Elena Efremova

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One of the most intriguing applications of chaotic signals is wireless communication systems. Interest in this issue has not waned for many years. Here the methods of wireless ranging between the emitter of chaotic signals and receiver are experimentally investigated. Chaotic signal occupies the 3-5 GHz frequency band. The physical and technical aspects of ranging based on measuring of chaotic radio pulses time propagation through a wireless channel are considered. An estimation scheme for measurements of the pulse envelope parameters in the receiver is proposed and experimentally tested. It is shown that the centimeter accuracy can be achieved that is a good result for the real wireless channels.

Keywords: Chaotic oscillators, chaotic ultrawideband signals, chaotic radio-pulses, wireless ranging, chaotic signal reception.

## The St Elmo's fire: Its formation and measurement on both manmade structures, as well as on insects

### V J Law, D P Dowling

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Since the beginning of recorded history Saint Elmo's fire (SEF) has been observed and experienced by humans in and close to thunderstorms which are dissipating in strength, either at sea level or in mountainous regions. The systematic study of SEF in nature has proved difficult due to inadequate measurement equipment and the understanding of highvoltage circuit requirements for partial gas breakdown measurement. The understanding of how SEF is generated is not only important in developing protection systems for shipping and aircraft flying at high altitudes and landing, but also for bioelectronics. Arguably our understanding of SEF is largely based on historical events and the empirical mathematical construct of Peek's Law, which attempts to identify the visual inception voltage in terms of the minimum electrical field stress required for the generation of corona discharge at sharp protrusions. This paper examines how SEF is formed around water-ice particles (graupel), as well on the surface of dirigible airships and airplanes. The paper also compares these 'natural' mechanisms to those which are recreated under laboratory conditions, using high-voltage direct-current, (HVDC-), high voltage alternating-current HVAC (50 Hz), as well as using a Tesla coil (0.3 to 30 MHz). SEF has also been investigated for the stimulation of living insects, it can also be used for their eradication. The Tesla coil circuit has been demonstrated to be suitable for the creation of a minimum visual inception voltage on both these living insects (moths and beetles), as well as on larger artificial objects such as ship models.

**Keywords:** St Elmo's fire, plasmoid, Aircraft surface protrusion, Insects, Tesla coil.

## Application of microwave oven plasma reactors for the formation of carbon-based nanomaterials

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This paper reviews the literature on the formation of carbon-based nanomaterials using converted domestic microwave oven plasma reactors. The carbon-based compounds range from single and multiwalled carbon nanotubes, to onion-like nanostructures, fullerene, and graphene sheets. The microwave plasma process is performed using inliquid containing plasma bubble (plasmoids) generated at an aerial-antenna igniter, susceptor surface ignition within gaseous plasma as well as the use of conventional gaseous plasmas. Based on the literature reports, the thermodynamic and kinetic plasma processing conditions are reviewed, along with process input criteria that include: applied microwave, hydrocarbon precursor, aerial-antenna igniter design, and susceptor material and sample collection. The use of microwave oven drilling (local thermal runaway) and reverse drilling that lead to the ejection of plasmoids that form dusty plasma or fireball is also reported for the formation of nanomaterials.

**Keywords:** Aerial-antenna igniter, Plasmoids, Ball-lighting, Fireball, Microwave oven, Carbon-based nanoparticles.

## Hair-pin Vortex Formation Mechanisms based on LXC-Liutex Core in Thermal Turbulent Boundary Layer with Rib tabulator

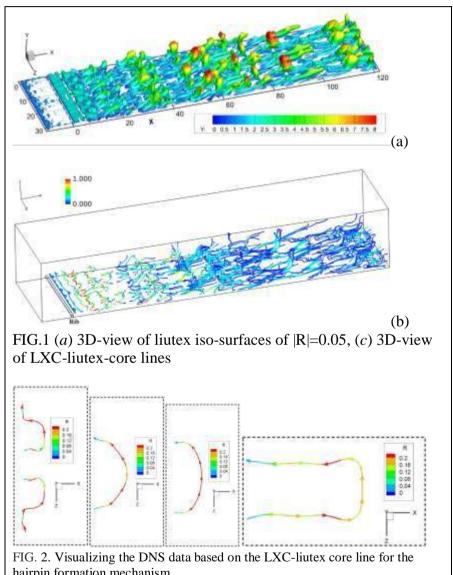
### Heng Li, Duo Wang, Hongyi Xu

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The three-dimensional flow fields were computed along a flat plate with rib tabulator by direct numerical simulation (DNS). The inlet Reynolds number (Re) defined by rib height was at Re=1200 and Mach number (Ma) was at Ma=0.2. The detailed bypass transition process and its generation mechanisms through the patterns of hairpin vortex are discussed and analyzed. The relatively larger skin-friction and pressure coefficients are found in the bypass transition. Moreover, the Reynolds stresses and turbulence kinetic energy are stronger in the transition, giving rise to the enhanced heat-exchange and therefore a larger Nusselts number (Nu). Various large-scale vortices were found downstream, where the momentum thickness Re was in the range of 250 < Re $\theta$  < 438. These large-scale vortices include the streamwise leg-shaped, arch-shaped, ring-like and hairpin vortices etc. The hairpin vortex was found originating from a spanwise vortex. Besides, a younger hairpin vortex can be generated from the primary hairpin vortex, a phenomenon captured by the current DNS. More importantly, the research discovers a new mechanism of a hairpin vortex being evolved from a pair of separated arch-shaped vortices. The current DNS solution presents the evidence for the generation mechanism with the convincing proof through visualizing both LXC-liutex iso-surfaces and core lines, i.e. the third-generation vortex identification as seen in FIG.1. Moreover, the dynamic evolution is quantified by the volume-averaged spanwise rotational strength defined by the scalar of liutex magnitude. The inherent mechanisms are analyzed based on the Biot-Savart law. It is concluded that, the new hairpin vortex is dynamically generated from a pair of arch-shaped vortices when the pair moving downstream side by side as seen in FIG. 2. In addition, the strongly-coupled correlations between the velocity and temperature

streaks are discovered and quantified for the first time, which is attributed to the well-known ejections and sweeps motions.

Keywords: Vortex identification, Liutex core, , Hair-pin vortex, Direct Numerical Simulation.



hairpin formation mechanism

## A Classification and criterion of Vortex Boundary based on Eigenvector

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National Science and Technology Major Project (2017-II-0006-0019, 2017-I-0009-0010)

It's known that Cauchy and Stokes gave that "the variation of velocity at each point is a superposition of a stretch along three perpendicular spindles (which can be decomposed into uniform expansion and pure deformation) and rigid rotation around an axis." In this paper, the concept of deformation is further divided into unidirectional shear and pure symmetric deformation. By using the principle of total shortest path, the decomposition of Cauchy and Stokes is extended to: "the variation of velocity at each point is a telescopic along three perpendicular spindles, superimposed with a rigid rotation around an axis or unidirectional shear. Then the classification method and discriminant conditions satisfying the vortex boundary are given, and it is emphasized that the unidirectional shear motion is an independent physical process. By discussing the eigenvector of the velocity gradient tensor, the boundary of the vortex in nature is divided into three cases: the broken type boundary, the continuous boundary and the mixed boundary. The boundary is displayed in the DNS results of the flow around the boundary layer of the plate. Keywords: velocity decomposition, vortex boundary, unidirectional shear process.

## Characterization of the Dynamic Behavior of the Net Present Value (NPV) in the Oil Market

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This paper studies the feasibility of investment in a petroleum reservoir. considering that the endeavor of this size requires a capital investment of high values. Thus, this work aims to evaluate the costs of the project by analyzing the net presente value (NPV) in function of the future price of oil and the productive capacity of the reservoir. Tho that end, this study is based on the investigation of the nonlinear characteristics of time series obtained by calculating the NPV whose verification is based on the methods of analysis of the presence of chaos through classical and novel techniques (machine learning) and allows us to define whether the investment is viable or no.

Keywords: Oil market, net present value, nonlinear dynamics, chaos, machine learning.

## Eigenfunctions of the Perron-Frobenius operator for uniformly hyperbolic area-preserving maps

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In hyperbolic systems, the largest subunitary eigenvalue of the Perron-Frobenius (PF) operator gives the rate of decay of correlations of dynamics. The associated eigenfunction is smooth along unstable directions whereas it behaves erratically in stable directions. Almostinvariant sets are dominant geometric structures related to the decay of correlations, that can be computed by mapping eigenfunctions of the PF operator backward in time through the Koopman operator. In this talk, we will discuss the relationship between almost-invariant sets and unstable periodic points in uniformly hyperbolic area-preserving maps.

Keywords: hyperbolic maps; Perron-Frobenius operator; Koopman operator; almost invariant sets.

**Liutex and Third Generation of Vortex Identification** 

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Vortex is a natural phenomenon with rotation everywhere in the universe. However, vortex has no mathematical definition for centuries. In almost all textbooks, vorticity is considered as the vortex strength since Helmholtz (1858). However, an immediate counter-example is laminar boundary layer where vorticity is very large near the wall, but vortex is zero (no rotation). The other counter-example is Poiseuille flow in a channel, which has analytic solution of  $u(y)=(1+y)(1-y), y \in [-1,1]$  where vorticity is very large near y=±1 but vortex is zero. Vorticity is classified as the first generation of vortex identification which cannot be used to represent vortex. In past three decades, many vortex identification criteria have been proposed like  $Q,\Delta,\lambda_2,\lambda_c$ i, which are completely determined by eigenvalues of  $\nabla v$ . They are all scalars, have no vector form, strongly threshold-dependent, and, more seriously, contaminated by stretching and shearing (mistreat stretching and shearing as vortex strength). These methods are classified as the second generation of vortex identification. They in general cannot represent vortex. Starting from 2014, the vortex and turbulence research team at University of Texas at Arlington have been working on new generation of vortex definition and identification. In 2016, an Omega vortex identification method was proposed to show the relative strength of vortex, which is threshold-insensitive. In 2017-2018, a Liutex vector was proposed to represent the rigid rotation of fluid motion, or vortex. Liutex has its scalar, vector, and tensor forms. Vorticity should be decomposed as a Liutex part and an anti-symmetric shear part (RS decomposition of vorticity). Velocity gradient tensor should be decomposed as a rotational part (Liutex) and a non-rotational part (R-NR decomposition) to replace Cauchy-Stokes decomposition. Meanwhile, the computation codes of Omega, Liutex, Omega-Liutex, Modified Omega-Liutex, Liutex core, etc. have been developed by the UTA team and published in the UTA Web Site at https://www.uta.edu/math/cnsm/public\_html/cnsm/cnsm.html for free download, which are classified as the third generation of vortex definition and identification methods. Vortex has six elements which only the third generation can represent these elements but not the first or second generation: 1. Vortex absolute strength: Liutex; 2. Vortex relative strength: Omega-Liutex; 3. Local rotation axis: Liutex direction; 4. Global vortex axis: Liutex core center lines; 5. Vortex core size: Omega-Liutex=0.52-0.6; 6. Vortex boundary: Liutex≥threshold>0. Vortex is a building block of turbulence. As a new physical quantity defined to represent vortex, Liutex will open a new era for vortex science and

turbulence research – from qualitative research to quantitative research. **Keywords:** Liutex, Vortex, Turbulence.

## Simulation and Analysis of Breaking Waves in Deep Water

### Cheng Liu, Yiding Hu, Decheng Wan

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Breaking waves has often been observed in ocean, the violent mixture of gas and water creates numerous bubbles and droplets in different scales. The generation of two-phase turbulence during breaking process accounts for a large portion of wave energy dissipation and exchange, which plays an important role in developing long-range weather model. However, the multi-scale turbulence of wave breaking has rarely been studied by experimental measurement and numerical simulations. In this study, the two-phase Navier-Stokes equations are solved based on Cartesian grid solver. For better resolution of the free surface and multiscale flow structures, a block-structured adaptive mesh refinement strategy is adopted. For better representation of the violent free surface, a mass-conservative interface capturing method CLSVOF (coupled levelset and volume of fluid) is implemented. Since a deep water assumption is hold, the viscosity of water and air will not be considered in current stage. For capturing the detailed flow characteristics of the plunging, a novel Liutex (previously named Rortex) is adopted for visualization of the vortex structure. A series of deep water breaking-wave cases considering various wave steepness (corresponding to weak plunging, plunging, strong plunging etc.) are under numerical investigation. The air entrainment, violent two-phase mixture during the breaking process are well reproduced. The time evolution of void fraction, turbulence kinetic energy and total energy dissipation are obtained from these numerical predictions. Finally, vortex structure captured by Liutex, Q-criterion etc. are given for comparison.

**Keywords:** wave breaking, deep water, large scale parallel simulation, adaptive mesh refinement.

## Breather interaction: Chessboard-like interference patterns and modulation instability

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Waves oscillating on a plane wave background known as breathers have received much attention in nonlinear science over the past decade. They play an important role in describing a variety of physical phenomena such as modulation instability, supercontinuum generation, turbulence, Fermi-Pasta-Ulam recurrence, and rogue wave events. In this talk, we present two scenarios of breather interaction, i.e., the chessboard-like interference pattern and modulation instability excited from localised initial perturbations. For the former, we show that the chessboard-like interference patterns exhibit rich structures and can be excited from various initial conditions. For the latter, we show recent progresses on super-regular breathers in nonlinear systems with self-steepening effect. **Keywords:** Breather, Interference pattern, Modulation instability.

### Visualizing Languages as Networks of Meaning

#### **Malcolm David Lowe**

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The conventional belief in modern linguistics is that the relationship of sound to meaning is fundamentally arbitrary. This assumption is false and as a result little or no progress has been made uncovering the true nature of languages. When investigated on the merits, meaning in a language can be shown to be part of a holistic system (of meaning) with a precise beginning, a distinctive architecture and a growth trajectory analogous to other living organisms. The difference between a meaning system and its biological counterpart lies in the fact that whereas the latter grows by the division and subdivision of cells, the meaning (or thought) system of a language grows by an iterative process of bifurcating concepts, beginning with the concept 'One'. Such systems can be represented as three-dimensional networks of interrelated and hierarchically ordered meaning relations that, while dynamic, always retain the same underlying architecture. Using the English language as a model, this paper focuses on how the system emerges, the gross design of the meaning system and how recurrent patterns in words can be used to understand the underlying architecture of the system.

**Keywords:** Languages as complex systems, meaning and network theory, modeling languages as systems, self-organization of meaning systems, architecture of meaning, arbitrariness of the sign, bifurcation of concepts.

## Two Types of Consciousness and Phase Transitions between Them

#### **Ihor Lubashevsky**

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First, I discuss common features of three basic concepts concerning human consciousness and perception being subjects of ongoing debates up to now. There are:

- The global workspace (GW) introduced by B. Baars to emphasize the integrative nature of consciousness. Then turning to the gist of GW N. Block singled out two types of consciousness—phenomenal (P) and access (A) consciousness—to discriminate between properties just experienced and properties resulting from rational analysis.
- The dual-process theory developed by J. Evans, D. Kahneman, and others which posits the existence of two systems—automatic, fast System 1 and deliberate, slow System 2—governing cognition, categorization, decision-making, etc.
- Inner psychophysics put forward by G. Fechner (1860) assuming the psychophysical laws are rooted in the brain functioning rather than reflect the individual properties of sensory modalities, which is related directly to the GW-concept.

Their common feature is the existence of two different, universal subsystems (phases, types) emerging via their own mechanisms, possessing individual stable properties, and governing complex mental phenomena in close interaction with each other.

In many physical systems exhibiting similar properties there have been found various phenomena of phase transitions. It has prompted us to search for phase transitions in mental phenomena and, to be specific, we focused on binary categorization which exemplifies characteristic aspects of all the three concepts.

So, second, I summarize the results of our experiments on binary categorization of (i) gray color, (ii) speech sounds, and (iii) number discrimination (preliminary results). Data analysis is based on constructing psychometric functions and focusing on asymptotics. The presented results enable us to posit the following.

- The choice between the auditory and visual categories as well as abstract categories in number comparison is governed by a universal central mechanism of the potential type. It is reflected in the linear asymptotics of psychometric functions in log-normal scales, which corresponds to significant uncertainty in categorization.
- There is a sharp transition between the linear asymptotic behavior of psychometric functions and their behavior when the choice of appropriate category is obvious. We relate the linear asymptotic behavior to conscious choice governed by slow System 2; it is characterized by the regular growth of decision time as the choice uncertainty increases. The choice of appropriate category when it is obvious seems to be governed by automatic, fast System 1.
- When the choice uncertainty is high, mental processes seem to affect substantially the decision time and its maximum can be used to quantify the relative contribution of conscious and unconscious processes. **Keywords:** P-Consciousness, A-Consciousness, Dual-Process theories, Binary categorization, Phase transitions.

## Principle of reactive-decision making in urban recovery after disaster under uncertainty of resilience boundaries

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The present research is devoted to the problem of socio-technical system resilience after disaster or another event significantly damaging it or decreasing the capability to perform properly. A modern city is represented as a complex structure including several layers of lifelines and thousands of individual actors. There are several approaches enabling to manage the recovery process: based on predetermined plan, based on the resilience triangle optimization and on the principle of selfregulation. The last one may be called as a reactive based decision making principle that enables to initiate the recovery process even in the case of informational insufficiency. This method is very useful for the shortterm recovery phase, the first part of system's recovery that should be started almost right after the disaster occurrence. In the given work attention is focused on the problem of boundaries estimation (minimal and maximal values) of resilience triangle that troubles initiation of mathematical approach of the recovery optimization. The main factors of this problem are: the impossibility to define the initial level of damage right after disaster and difficulty to estimate the minimal standard of system's performance satisfying the surviving level. Without knowledge of this two points the mathematical approach to solve the nonlinear problem of the system recovery is inapplicable. The proposed solution to this problem is split into two parts. The upper boundary of resilience triangle is defined over agent based modelling of lifeline consumption based on the statistical data about previous consumer's behavior and on prediction of the agent's behavior during the city restoration period. The lack of the initial information is avoided by using the nature of the city (distributed multiagent complex system) and the usage of the reactive decision making based recovery principle. As a result, the recovery process might be initiated in hours after the disaster occurrence, be based on the local/inner resources of the damaged urban area and being guasi optimal.

**Keywords:** Complex system, Agent based modeling, Disaster resilience, Adaptive process.

## Traveling waves and spatio-temporal chaos in nonlinear partial differential equations

### Nikolai A. Magnitskii

Federal Research Center "Computer Science and Control" of RAS, Lomonosov Moscow State University, Moscow, Russia It is considered some nonlinear partial differential equations having physical, chemical or biological turbulence regimes, such as the Ginzburg-Landau, Kuramoto-Sivashinsky, Schrödinger, FitzHugh-Nagumo equations. It will be shown, that all such systems of partial differential equations can have an infinite number of different stable wave solutions. travelling along the space axis with arbitrary speeds, and also an infinite number of different states of spatio-temporal chaos. These chaotic (turbulent) solutions are generated by cascades of bifurcations of cycles or tori and singular attractors according to the universal bifurcation Feigenbaum-Sharkovsky-Magnitskii (FShM) theory in the threedimensional or four-dimensional systems of ordinary differential equations, to which the systems of partial differential equations can be reduced by self-similar change of variables. Examples of application of the theory to the description of processes of physical, chemical and biological turbulence will be considered.

**Keywords:** Nonlinear PDE, Traveling waves, Spatio-temporal chaos, Turbulence, FShM-theory

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## Study of turbulent transport in magnetized plasmas with flow using symplectic maps

#### Julio J. Martinell, Nikolay Kryukov

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Test particle E X B transport due to a discrete spectrum of drift waves in two dimensions is studied using a Hamiltonian approach, which can be reduced to a 2D mapping. Finite Larmor radius (FLR) effects are included taking a gyroaverage. The presence of poloidal flows is included which can give rise to transport barrier formation. For large wave amplitudes there is a transition to chaos and the barriers are destroyed. FLR effects tend to restore the barrier, implying that fast particles are better confined. For a thermal FLR distribution, the PDF is non-Gaussian while the transport remains diffusive when there is no flow but becomes ballistic when the flow is strong enough. When the background flow varies linearly with radius, the single-step map can be symplectic but for more general flows a two-step map should be used. The stability of transport barriers is explored for several types of flow.

**Keywords:** Hamiltonian chaos, Transport barriers, Magnetized plasma transport, Non-twist maps.

## On the Exterior Biharmonic Problem with the Steklov and Steklov-type Boundary Conditions

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We study the unique solvability of the mixed biharmonic problem with the Steklov and Steklov-type conditions on the boundary in the exterior of a compact set

under the assumption that generalized solutions of this problem has a bounded Dirichlet integral with weight  $|x|^a$ . Depending on the value of the parameter \$a\$, we obtained uniqueness (non-uniqueness) theorems

**Keywords:** Biharmonic Operator, Steklov and Steklov-type Boundary Conditions, Dirichlet Integral, Weighted Spaces.

## Biharmonic Problems and their Applications in Engineering and Technology

### Hovik Matevossian<sup>1,2</sup>, Giorgio Nordo<sup>3</sup>, Giovanni Migliaccio<sup>4</sup>

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We study some properties of solutions of biharmonic problems. Namely, we study the Steklov, Steklov-type and Neumann boundary value problems for the biharmonic equation. For solving these biharmonic problems with application, in particular, to radar imaging, we need to solve the Dirichlet, Neumann and Cauchy boundary value problems for the Poisson equation using the scattering model. In order to select suitable solutions, we solve the Poisson equation with the corresponding boundary conditions, that is, some criterion function is minimized in the Sobolev norms. Under appropriate smoothness assumptions, these problems may be reformulated as boundary value problems for the biharmonic equation. **Keywords:** Biharmonic Operator, Boundary Value Problems, Scattering Model, Variational Method.

## Estimation of Ergodic Maps with Unified Power Spectra from Causal Sequences of State Density Functions

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<sup>1</sup>Council for Scientific and Industrial Research, Pretoria, South Africa, <sup>2</sup>University of the Witwatersrand, Johannesburg, South Africa In this paper, we consider the problem of modelling the evolution rule of a deterministic, one-dimensional dynamical system from observations corrupted by noise. We consider a scenario where individual trajectories of the system state cannot be observed directly, as is encountered in application areas such as particle image velocimetry for flow visualisation and flow cytometry. In this scenario, conventional techniques for reconstructing the system's evolution rule from noise-corrupted time series data, such as neural networks and wavelet models, cannot be applied. However, it is often possible to observe how density functions of the system's state evolve over time, and, from these sequences of density functions, attempt to construct a map with statistical characteristics that approximate those of the unknown evolution rule. Several solutions to the above problem were proposed recently. These solutions, which assume that the unknown system is ergodic, model the evolution rule as a piecewise-linear (PWL) Markov map. The evolution of piecewise-constant (PWC) density functions under a PWL Markov map is fully characterised by a matrix representation of the map's Frobenius-Perron Operator (FPO). The modelling problem is solved by fitting PWC functions to the observed state densities and computing an estimate of the matrix that corresponds to the unknown map's FPO from these functions. The Markov map is subsequently constructed from the matrix estimate. Numerical experiments have shown that this approach can, in certain cases, produce a model with an invariant state density which approximates that of an unknown map. However, these techniques do not necessarily produce models where the correlation function and power spectrum of state trajectories approximate those of the unknown system. Hence, the present technique is unable to model important characteristics such as the average time the system requires to settle into statistically regular behaviour, and the average rate of trajectory divergence.

We show that a second-order matrix representation of a PWL Markov map's FPO, which governs the evolution of PWL density functions (i.e., instead of PWC density functions) characterises the correlation function and power spectrum of the map, in addition to the invariant density. A novel subset of the PWL Markov maps, referred to as generalised hat maps, is defined; it is shown that the structure of these maps facilitates the selection of the second-order matrix representation of the corresponding FPO. We propose a novel technique for estimating the second-order matrix from sequences of observed PWL density functions, and constructing a generalised hat map with a matching second-order matrix representation of the FPO. Numerical examples are presented

which show that the models obtained using the proposed technique have power spectra and invariant densities that closely approximate those of several cases of piecewise-defined maps. We conclude that the new technique allows for more accurate and realistic modelling of unknown one-dimensional deterministic systems.

Keywords: Chaotic maps, ergodic maps, inverse Frobenius-Perron problem, nonlinear systems, probability density functions, correlation function, power spectrum.

### On the complexity of the q-deformed logistic map

#### María Muñoz-Guillermo

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We compute topological entropy of the a-deformed logistic map in order to analyze the complexity of the dynamics of the q-deformed logistic map in the parametric space. In fact, we are able to prove the existence of topological chaos. This will be used to analyze the nature and the coexistence of attractors.

This is a joint work with J. Cánovas (Universidad Politécnica de Cartagena, Cartagena, Spain)

**Keywords:** Topological entropy, Chaos, q-deformations.

## Introduction to identification and control of thermonuclear plasmas

### Andrea Murari<sup>1</sup> and JET Contributors\*

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Next generation of Tokomak devices will have to make a much more systematic use of feedback control than present day machines. The higher energy content, longer pulses and increased sophistication of the magnetic configurations are indeed all elements expected to render traditional feed forward schemes highly inadequate. The systematic use of DT as fuel will also require to implement more robust safety precautions. In this context, successful real time strategies will have to rely on proper identification of the plasma and on detailed understanding of the main instabilities, with the potential to seriously affect the evolution of the discharges or the integrity of the devices. The previous considerations have motivated an increasing interest in new techniques to solve inversion problems for identification, particularly equilibrium reconstructions and tomographic inversions. With regard to the instabilities, in addition to new approaches to disruption prediction, a series of causality detection and correlation tools are being developed particularly for the investigation of triggering experiments o ELMs and sawteeth.

\*See the author list of E. Joffrin et al. accepted for publication in Nuclear Fusion Special issue 2019.

https://doi.org/10.1088/1741-4326/ab2276

## Fractal Dimension of Braided Rivers from Detailed Two-Dimensional Hydrodynamic Simulations

#### **Matteo Nicolini**

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As opposed to meandering or channelized fluvial beds, braided rivers are characterized by a morphological activity starting at very low flows, since fluxes are concentrated in a limited number of small channels. With increasing discharge, more channels are involved, up to the situation in which the complete alluvial plain is flooded. As a consequence, there is an intermediate range of flows for which pattern complexity is maximum and braided indices are highest, representing essential conditions for the coexistence of a large variety of habitats and for ecosystems prosperity. In this paper, a new methodology for a quantitative assessment of the complexity of braided rivers at a reach scale is introduced. It is based on the application of the box-counting algorithm to flooded areas as identified with a two-dimensional (shallow water) hydrodynamic simulation model, in order to derive an estimate of the fractal dimension with varying flow rate. The identification of the range of discharges for which the fractal

dimension is highest is of particular importance in river restoration projects. An application to the River Tagliamento (North-East Italy) is illustrated.

**Keywords:** Morphodynamics, River restoration, Formative discharge, Box-counting algorithm.

## Visualization of the Batchelor Vortex with Liutex and Liutex Core Line Methods

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Many researchers have spent a lot of efforts in the research of the identification of vortices for several decades. These efforts have produced many methods allowing us to better identify and visualize vortices. The most popular traditional methods in literature are Q,  $\Delta$ ,  $\lambda_2$  and  $\lambda_{ci}$  with Q being the most popular method used in research and industry. In this paper, the new methods of Liutex and Liutex Core Line are used to identify and visualize the Batchelor Vortex. A comparison of these methods with the Q method is performed and analyzed. The results of the comparison emphasize the superiority of the Liutex based methods over the traditional and popular method Q.

**Keywords:** Vortex, Batchelor Vortex, Liutex, Liutex Core Lines, Visualization.

## The topological Hausdorff dimension and improved oil recovery on Menger sponge and Cantor Tartan

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We study how fractal features in a permeable material influences its transport properties, particulary the improved oil recovery.

The fractal attributes are characterized by the Hausdorff, topological Hausdorff and topological connectivity dimensions. Improved oil recovery processes were simulated on infinitely ramified fractals (Menger sponge and Cantor tartan) with different fractal attributes. Our finding suggest that recovered oil is well-described by the set dimension numbers dH, dtH and dlt rather than solely mass fractal dimension. We also argue that the revelant dimension number for the reservoir Cantor-tartan-like is the Hausdorff dimension dH, whereas the scaling exponent on a fractal matrix type Menger sponge is governed by the topological Hausdorff dimension dtH.

**Keywords:** Topological Hausdorff dimension, Topological connectivity dimension, Cantor tartan, Menger sponge.

## Turbulent transport control by tokamak plasma rotation

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The direct effects of the toroidal and poloidal plasma rotation on transport are analysed in the frame of a test particle model based on a realistic description of the turbulence [1]. The results apply to impurity and fast particle transport [2]. We show that the toroidal velocity Vt can influence the transport only through the modification of the decorrelation time  $\tau d$ . However, the change of the transport coefficients strongly depends on the transport regimes and significant differences appear between the types of the advected particles (light and heavy impurities, or fast  $\alpha$  particles). The effects of the poloidal rotation Vp are much more complex. Essentially, there are two physical processes that influence the transport. One is the modification of the configuration of the contour lines of the stochastic potential by the average potential rVp. The radial displacements on the contour lines are limited due to the strips of open contour lines that are generated. This leads to the decrease of the transport coefficients that is controlled by the value of Vp. A more subtle effect of poloidal rotation is

the generation of the hidden drifts [HDs], which are ordered radial components of the ion motion with opposite directions that exactly compensate one another [3]. The polarization drift (or the parallel acceleration) perturbs the symmetry of the HDs and determines a radial convective transport Vr [4]. Compared to the diffusive flux, the convection with the velocity Vr can be dominant for heavy impurities.

The dependences of the diffusive and convective transport on the rotation velocities Vt and Vp are obtained as functions of the parameters of the turbulence and of the characteristics of the particles using semi-analytical theoretical methods. The results are validated by comparison with direct numerical simulations. The conclusion of the study is that the rotation velocities provide efficient control parameters for the turbulent transport.

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## **Determination of chaotic behaviour in time series** generated by charged particle motion

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We use non-linear methods of chaos detection to investigate the behaviour of ionized region of a Keplerian disk orbiting a Schwarzschild black hole immersed in an asymptotically uniform magnetic field. In order to study transition between the regular and chaotic type of the charged particle motion, we generate time series of the solution of equations of motion under various conditions, and study them by non-linear (box counting, correlation dimension, Lyapunov exponent, recurrence analysis, machine learning) methods of chaos determination. We demonstrate that the machine learning method appears to be the most efficient in determining the chaotic region. In dependence on the magnetic parameter B, and inclination angle  $\theta$  of the disk plane with respect to the magnetic field direction, the charged particles of the ionized disk can enter three regimes: a) regular oscillatory motion, b) destruction due to capture b! y the magnetized black hole, c) chaotic regime of the motion.

**Keywords:** black holes, charged particle motion, chaos and regularity, Lyapunov exponent, correlation dimension, machine learning.

### Supergranulation - A Chaotic Phenomenon

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Supergranulation is examined through its various parameters such as area, perimeter, horizontal flow velocity, lifetime, circularity and fractal dimension. The chaotic nature of supergranulation is established in its turbulent origin. An interrelationship amongst the various parameters of these convective eddies are in support of this behavior. These findings are supportive of Kolmogorov's theory of turbulence.

Keywords: Sun; Granulation - Sun; Activity - Sun; Photosphere.

### Chaotic regimes in Josephson junctions and chains

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We report about various chaotic phenomena, observed in Josephson junction chains and in high-temperature (HTSC) Josephson junctions. In modeling we consider complex dynamics with interplay between chaotic regimes and thermal noise and demonstrate that chaos can significantly increase the spectral linewidth, which is important for Josephson junctions application as THz oscillators [1]. HTSC long Josephson junctions demonstrate diverse nonlinear dynamics. Using samples, fabricated in IPM RAS, the current-voltage characteristics have been measured for various magnetic fields and temperatures in the presence of external ac driving. The Shapiro steps with a number of fractional

substeps have been observed and their appearance is explained by quasi-chaotic dynamics of the weak Josephson soliton chain at low magnetic fields, which is confirmed by the numerical solution of the sine-Gordon equation and demonstrated that fractional steps disappear at larger magnetic fields, making the vortex chain stronger.

The work was supported by the Russian Science Foundation (project 16-19-10478).

Keywords: Josephson junction, chaotic regimes, noise.

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## Correlation Analysis between low frequency shock oscillation and Liutex in SBLI

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Low frequency oscillations of shock and turbulent boundary layer interaction (SBLI) are very harmful and major obstacle in design and flight of supersonic commercial aircrafts due to the unacceptable low frequency noises. To find the source of the low frequency noise and SBLI control, researchers have spent almost one century to find the mechanism but with fruitless outcome. It is still a mystery that the low frequency noise of SBLI is caused by inflow turbulence, shock induced boundary layer separation or others. After several years of research, Liu et al have found that the SBLI is really shock-vortex interaction and the noise is overwhelmingly caused by shock oscillation and the low frequency noise is determined by the vortex size and moving speed. Dong et al found that using micro vortex generator (MVG) can reduce the shock oscillation and then remove the low frequency noise but keep the high frequency oscillations which are less harmful. These important discoveries have paved ways for people to

understand the source of SBLI caused low frequency noise and pave way for commercial supersonic airplane to become reality. This paper is to confirm the low frequency shock oscillations is dominated by Liutex transportation through a correlation analysis between shock oscillation and Liutex. A detail analysis will be reported by the final paper.

**Keywords:** - SBLI, Micro-vortex generator, shock oscillation, Liutex transportation, correlation analysis.

## Numerical methods for the nonlocal wave equation of the peridynamics

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We consider the peridynamic equation of motion, which is described by a second order in time partial integro-differential equation. This equation has recently received great attention in several fields of Engineering because seems to provide an effective approach to modeling mechanical systems avoiding spatial discontinuous derivatives and body singularities. In particular, we consider the linear model of peridynamics in a one-dimensional spatial domain. We review some numerical techniques to solve this equation and propose some new computational methods of higher order in space. We discuss the implementation of a spectral method for the spatial discretization of the linear problem. Several numerical tests are given in order to validate our results.

**Keywords:** Peridynamic equation, Quadrature formula, Spectral methods, Trigonometric time discretization.

Assessing the effectiveness of synchronisation experiments in magnetic confinement fusion: information theoretic and recurrence criteria

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The control of instabilities such as ELMs and sawteeth is an essential element in the development of reactor relevant scenarios. Various forms of ELM pacing have been tried in the past using external perturbations. Sawteeth synchronization with ICRH modulation is a recent approach to influence the behaviour of these instabilities. One of the main issues of all these synchronization experiments resides in the fact that the instabilities are quasi periodic in nature and therefore, after any pulsed perturbation, if enough time is allowed to elapse, they always occurs. To evaluate the effectiveness of the various pacing techniques, it is therefore essential to determine an appropriate interval over which they can really have a triggering capability. In this paper, a methodology based on various independent statistical methods, belonging to four main families (Granger Causality, Transfer Entropy, Convergent Cross Mapping and Recurrence Plots), is described to address this issue. The obtained results for JET with the ILW indicate that the proposed techniques agree very well and provide much better estimates than the traditional heuristic criteria reported in the literature. Moreover, their combined use allows providing confidence intervals in the conclusions. Therefore, the developed method can be used to provide a quantitative and statistically sound estimate of the triggering efficiency of pacing methods in realistic experimental conditions and to support theoretical explanations of the phenomena.

\*See the author list of E. Joffrin et al. accepted for publication in Nuclear Fusion Special issue 2019,

https://doi.org/10.1088/1741-4326/ab2276

### Edge Localised Modes: evidence of chaotic dynamics?

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Edge localized Modes (ELMs) are plasma instabilities occurring in the edge region of a tokamak plasma [1]. Their intermittent and aperiodic behavior has led several researchers to investigate about their dynamical characterization, obtaining results sometimes conflicting. In this work, the dynamical characterization of type-I ELM time series data from the JET tokamak, the world's largest magnetic confinement plasma physics experiment, is discussed, in order to understand the dynamic origin of ELM behavior. The discussion starts from the results obtained in [2], where it's concluded that the ELM phenomenon is consistent with the presence of a dynamic nonlinearity corrupted by noise, i.e. a pseudoperiodic behavior. This characterization has an implication on the dynamical models which are supposed to reproduce the ELM dynamics. In this regard, a modification of a periodic state-of-the-art model [3], able to reproduce the ELM cycle, is also proposed.

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https://doi.org/10.1088/1741-4326/ab2276

## Opportunities for using strip and fractal antennas based on chiral metamaterials in 5G networks

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An important task in mobile radio communication networks is improving of the service quality by applying an integrated approach to modernizing the entire system as a whole, since currently, due to the availability of multimedia content high-quality, bandwidth requirements are becoming more stringent. Along with all improvements in the network software, for example, the using of a new signal-code constructions, various types of modulation (N-OFDM), non-orthogonal multiple access (NOMA) technologies, etc., there is a need the hardware improving. One example of the implementation of this approach can serve as a network of new generation 5G [1], allow to providing greater bandwidth compared to previous generations networks. The key technology of 5G networks is Massive MIMO technology [2], the essence of which is the organization of many weakly correlated sub-channels of information exchange between the source and receiver, thereby significantly increasing the spectral efficiency of the entire system. From the point of view of antenna technology, the using of antenna systems new types, including those based on chiral metamaterials, can further improve the quality of service. Metamaterial is an artificial composite material with spatial dispersion. The peculiarity of metamaterials is that their effective permittivity and magnetic permeability can simultaneously take negative values. Among metamaterials one should single out chiral metamaterials, which are a set of mirror-asymmetric shape conductive inclusions which uniformly distributed in a homogeneous dielectric container [3]. It was found that the radio networks bandwidth using MIMO technology is quite dependent on the magnitude of the mutual influence of the antenna array elements. The using of antenna arrays based on chiral metamaterials allows in MIMO systems to increase throughput.

**Keywords:** chiral metamaterial, antenna, 5G, MIMO, fractal.

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# Fractal antennas based on biisotropic and bianisotropic chiral metamaterials

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Currently there is an active development of both radio electronics in general and antenna technology in particular. Today one of the most dimensional elements of modern portable devices is often an antenna. One of the promising directions in improving antenna technology is associated with the using of artificial composite materials called metamaterials in their designs. The metamaterials using in antennas allows improving their electrical and mass-dimensional characteristics [1-4], in particular, improving their directional properties, as well as to reduce dimensions, due to compensation of the reactive part of the input impedance. An example of this approach is the MIMO technology [5], which is widely used in mobile communication networks, including in the networks of the new generation 5G.

As is known, fractal antennas have multiband properties, i.e. able to work in several frequency ranges. In order to improve the characteristics of fractal antennas, it is of interest to analyze an antenna with a fractal emitter based on chiral metamaterials.

The Feko electrodynamic modeling software package was used for calculation the characteristics of the metamaterial antenna based on a set of helices. In addition, the metamaterial substrate resonant frequency and, accordingly, the helices geometry were also determined by using the optic activity media model dispersion [6]. Studies have shown that the

biisotropic and bianisotropic chiral substrates using can improve the fractal antennas characteristics.

**Keywords:** Microstrip antenna, chiral metamaterial, fractal. **References:** 

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# Construction of composite chaotic multiattractors containing local chaotic attractors with different spatial orientations

#### Vadim Prokopenko

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A method for constructing autostochastic dynamic systems with a multiattractor region of attraction of phase trajectories containing identical chaotic attractors with different orientations in the phase space of the system is considered. **Keywords:** dynamic chaos, generator of chaotic oscillations, composite chaotic multiattractor, heterogeneous multiattractor, replicate operator, phase cell.

# A symmetry analysis of differential systems on functional manifolds

#### Anatolij K. Prykarpatski

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This Report is devoted to some aspects of the symmetry analysis related with analytical techniques for constructing differential conervation laws to a general smooth functional-differential system on a functional manifold. The problem of existence and constructing conservation laws to differential systems goes back to classical past centures works of C. Jacobi, J. Liouville, J. Lagrange, S. Kovalevska, P. Painleve, H. Poincare, S. Lie and of others mathematicians, and culminates in the eminent E. Noether article (see: Noether E. Invariante Variationsprobleme, Nachr. d. Konig. Gesellsch.d. Wiss. zu Gottingen, Math-Phys. Klasse, 235-257, (1918); Transport Theory and Statistical Physics, 1(3), (1971), p. 183 on invariants of a general Lagrangian variational problem. It is really difficult to overestimate the importance of conservation laws taking into acount both their fundamental role in formulating of main Laws of Nature and their multiple practical applications in many areas of modern applied mathematics, physics, biology and economy. It is also well known their role in many domains of modern pure mathematics, including differential geometry, topology and integrability theory of nonlinear dynamical systems on manifolds. There are also many closely related to conservation laws \important mathematical problems, amongst which one can mention symmetry group classification, conditional symmetry analysis, inverse problem on group classification etc., whose detailed and meticulous analysis was performed in many determining new research areas works during the past century and during the last decades yet, by no means, claiming their completeness. In this Report there is successively sketched a classical E. Noether based analytical scheme of constructing the complete set of global conservation laws to a smooth vector field on a functional manifold. There is described in detail an analytical algorithm of constructing the complete set of differential conservation laws to a general smooth functional-differential system on a functional manifold with values in an arbitrary vector bundle. There is also demonstrated that the algorithm reduces, as a special case, to that presented recently by S.C. Anco in his recent work (see: Anco S.C. On the incompleteness of Ibragimov's conservation theorem and its equivalence to a standard formula using symmetries and adjoint-symmetries. Symmetry, 2017, 9(3), 33 p.)

**Keywords:** differential-function system, functional manifold, symmetry, adjoint-symmetry, Lagrangian, Hamiltonian, symmetry analysis, conservation law.

### Chaotic generator design for encryption purposes

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This paper is devoted to the synthesis of chaotic generators over a finite-state space. The specificities of the design requirements are discussed and analyzed from dynamical systems theory point of view. The motivation of the study arises from the fact that in cryptography, finite set of large integer numbers is used, whereas the great majority of well-known chaotic maps is based on real numbers. Consequently, when a chaotic generator based on real numbers is numerically implemented, the latter results in quantization errors and round-off errors, which may lead to security breach. For these reasons, chaotic generators based on real numbers turn out to be inappropriate for cryptography. In general, simulating chaotic orbits over integer levels with finite precision may give rise to degraded periodic orbits, where a stopgap solution would be to render these periodic orbits very long period ones.

In this paper we discuss the fundamental rules to design finite field chaotic generators for encryption purposes. Several well-known one-dimensional chaotic maps are reformulated over a finite-state space, and their dynamical and statistical properties are analyzed. Coupling of one-dimensional chaotic maps has already been suggested to enhance complexity and avoid numerical instability. However, a critical issue here is the correct choice of the couplings, which should be tuned carefully

since small increase or decrease in one of the coupling parameters may show big change of the overall nonlinear system dynamics. In particular, close coupling combinations under small variations may lead to very different results, and for some specific parameters the system can get locked into short orbits or even fixed points. Therefore, the range of coupling parameters (meaningful for encryption) must be finely scanned to identify the most appropriate couplings and avoid undesirable dynamical behavior. At the end, the designed chaotic generator over a finite field must satisfy all statistical tests for randomness while saving precious computational time, since nor quantization, nor truncation or round-offs are required.

**Keywords:** Chaotic map, Finite-state space, Coupling, Nonlinear dynamics, Periodic orbits.

### **Rogue Waves in Nonlinear Integrable Systems**

## Zhenyun Qin

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General higher-order rogue waves of a vector nonlinear Schrodinger equation (Manakov system) and Sasa-Satsuma equation are derived using a Darboux dressing transformation with an asymptotic expansion method. The N-th order semirational solutions containing 3N free parameters are expressed in separation of variables form. These solutions exhibit rogue waves on a multisoliton background. They demonstrate that the structure of rogue waves in this two-component system is richer than that in a one-component system. Our results would be of much importance in understanding and predicting rogue wave phenomena arising in nonlinear and complex systems, including optics, fluid dynamics, Bose-Einstein condensates and finance.

**Keywords:** Rogue waves, Integrable systems, Manakov equations.

## Visualization of Complex Flow Field of Ship Self-Propulsion and Zigzag Manoeuvrability

### Zhen Ren, Weiwen Zhao, Decheng Wan

State Key Laboratory of Ocean Engineering, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai, China Vortices are widely found in naval architecture and ocean engineering. Especially in wake field of ship self-propulsion, there are a large number of vortices with different sizes and strengths, which play a key role in the process of turbulent flow generation and maintenance. Therefore, accurate identification of vortices is very import to understand the flow mechanism in the wake field of ship self-propulsion.

During the last decades, there are many proposed vortex identification methods, including Q,  $\Delta$ ,  $\lambda$ 2 and  $\lambda$ ci criteria. However, these methods are based on the Cauchy-Stokes decomposition and/or eigenvalues of the velocity gradient. And these methods are not able to capture both strong and weak vortices simultaneously. In the present study, the third generation of vortex identification, liutex, is adopted to capture the vortex structure in the flow field of ship self-propulsion. The method can capture both strong vortices and weak vortices, and the six core issues for vortex definition and identification has been raised. At the same time, the second generation of vortex identification method, Q criterion, is applicated, and the results are compared with that obtained by using the liutex method. In the numerical simulations, the self-propulsion and zigzag manoeuvrability of KCS with a Lpp length of 6.0702m are carried out, couping with KP505 propeller and the rudder with NACA0018 section. And the Froude number is 0.26. The vortex structure in both cases are present. Firstly, the vortex structure in the self-propulsion is analyzed. Both resluts obtained by liutex method and Q criterion are compared. And the same comparison of calcluted results is carried out in zz20/20 manoeuvrability. The results show that the liutex method better presents the size and direction of the vortex structure.

**Keywords:** liutex, Q, vortex identification, self-propulsion, zigzag manoeuvrability.

# Information measures and synchronization in regular ring lattices with discontinuous dynamics

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1CEAUL. ADM, ISEL-Eng. Superior Institute of Lisbon, IPL, Lisboa, Portugal, <sup>2</sup>CEAFEL. ADM, ISEL-Eng. Superior Institute of Lisbon, IPL, Lisboa, Portugal We study information measures and synchronization in complete dynamical networks of maps, with local identical chaotic dynamical systems. The network topologies are regular ring lattices which are characterized by circulant matrices and the conditional Lyapunov exponents are explicitly determined. For discontinuous local dynamics, some properties of the mutual information rate and the KolmogorovSinai entropy are established, depending on the topological entropy of the individual chaotic nodes and on the synchronization interval. It is proved that as large as the network topology is, measured by its network topological entropy and directly related with the network order, the information measures studied increase or decrease, according to the network order in relation to the synchronization interval. Some numerical studies are included.

**Keywords:** Mutual information rate, Kolmogorov-Sinai entropy, synchronization, complete networks, discontinuous dynamics, Lyapunov exponents, topological order, circulant matrix.

## How chaotic dynamics drive a vintage grill-room spite

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In 1943 Yves Rocard published an amazing book entitled "Dynamique Générale des Vibrations". Among a collection of mechanical devices that are studied with great care in the book, the so-called Bouasse-Sarda's "tournebroche" (rotary grill spite) is a fascinating two degrees of freedom device. What makes it interesting is that it stylises a wide class of forced parametric oscillators exhibiting a rich range of complex dynamical behaviours including fully deterministic chaos, a dynamic concept not popular in the forties and hence not yet discussed in the Y. Rocard's book. Today's numerical tools offer a new possibility to revisit this amazingly simple but rich dynamical system.

**Keywords:** Nonlinear coupled mechanical systems, Stability analysis, "Washboard" like dynamics, Chaotic dynamics, Feigenbaum cascade, Period-doubling bifurcations, Antimonotonicity.

Nonlinear Analysis of El Niño-Southern Oscillation Events from Coastal Temperature Time Series in the

## **Eastern South Pacific: Implications for the Regional Bioclimate System**

## Berenice Rojo-Garibaldi<sup>1</sup>, Manuel Contreras-López<sup>2</sup>, Matías Carvajal<sup>3</sup>, David Alberto Salas-de-León<sup>4</sup>, Julvan H. E. Cartwright<sup>5,6</sup>

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In this work we study from the viewpoint of the theory of nonlinear dynamical systems the effect of El Niño on coastal temperatures on the eastern South Pacific coast, characterized by the Humboldt Current System (HCS). We show that classical linear trend analyses are not able to characterize the complex and chaotic temporal behaviour of the coastal temperature records exposed to the HCS; however, the nonlinear analysis indicates that the evolution of the temperature in the Eastern South Pacific littoral can be embedded in a 3-dimensional space. Through a nonlinear analysis of nine long temperature time series distributed between 18°S and 45°S, for periods before, during and after El Niño events of 1982/83 and 1997/98, we found that all stations studied along this transect show chaotic behaviour, presenting in general a larger positive Lyapunov exponent after this phenomenon, which indicates that the level of chaos is increasing in the HCS. This suggests that the bioclimate system in these regions is becoming more unpredictable in the conditions imposed by El Niño. However, in the case of three stations the level of chaos is lower during El Niño, from which we interpret that in these areas this may be a necessary phenomenon to stabilize the regional bioclimate system, associated with the decrease in phytoplankton and primary production.

Keywords: Chaotic dynamics, nonlinear times series, Lyapunov exponent, El Niño.

# Neural computation for the assessment of correlations between quantities in large multimachine databases

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The increasingly sophisticated investigations of high temperature plasmas necessitate more robust estimates of the correlations between the measured signals. The traditional Pearson Correlation Coefficient is easy to calculate but is sensitive only to linear correlations. An alternative, sensitive to the total influence between quantities, is the Mutual Information, which on the other hand is not normalised. To compare data from different experiments, the Information Quality Ratio is therefore of easier interpretation. Unfortunately, both the Mutual Information and the Information Quality Ratio are positive indicators and therefore cannot provide information about the sign of the mutual influence between quantities. From a practical point of view, one additional difficulty is that they require an accurate determination of the probability distribution functions of the variables involved. Since the amount of data available and the level of noise are not always sufficient to grant an accurate estimation of the probability distribution functions, it has been investigated whether neural computational tools can help and complement the aforementioned indicators. Specific encoders and autoencoders have been developed for the task of determining the total correlation between quantities, including information about the sign of their mutual influence. Their accuracy and computational efficiencies have been addressed in detail, with extensive numerical tests using synthetic data. The first applications to experimental databases have provided quite encouraging results.

# Role of the neuronal firing rate in emergence of chaotic brain extracellular matrix dynamics

Maiya A. Rozhnova, Victor B. Kazantsev, Evgeniya V. Pankratova Lobachevsky State University of Nizhni Novgorod, Nizhny Novgorod, Russia Recent experimental data have shown that the brain extracellular matrix (ECM) can play significant role in processing of external information in neuronal networks. The increasing interest to this subject led to development of mathematical formalism for exploration of its complicated impact on neuronal activity. As was shown recently, the complexity is in existence of feedback mechanisms of neuron-ECM interaction. Neuronal activity-dependent modification in concentrations of the ECM molecules affects the synaptic plasticity and defines crucial role in learning and memory [1]. On the basis of mathematical model introduced in [2] for homeostatic regulation of neuronal activity by ECM, for various constant neuronal firing rate levels, non-trivial bi-stable regimes of ECM dynamics was recently obtained in [3]. At the same time, consideration of full highdimensional model of neuron-ECM interaction revealed that firing rate level can be dependent on time. In this study, we examine the role of timedependent neuronal firing rate level in emergence of highly complicated ECM dynamics. Bifurcational mechanisms for the emergence of irregular oscillations of ECM molecules concentration described by various chaotic attractors are carefully studied. Its co-existence with the regular attractors in the phase space of the system is shown. Non-trivial multi-stabilities observed in ECM dynamics, disclose the opportunities for appearance of aberrant neuronal activity.

Keywords: Chaos, Brain extracellular matrix, Multistability.

The work was supported by the Ministry of Science and Higher Education of the Russian Federation (Project No. 14.Y26.31.0022).

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### Pulse chaotic Chua's generator

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Scheme technical realization with nominal values for the pulse transformation is presented. This circuit can be used for transform analog nonlinear signal to pulse. For shows of computer modeling results of this process was selected software MultiSim. Experimental results are also presented. The designed layout was applied for chaotic Chua's generator. **Keywords:** Chaos, Pulse, MultiSim, Chua's Generator.

# Software realization, analysis and experimental investigation of equivalent inductance

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Circuit realization of the inductance equivalent that contains two operational amplifiers, one capacitor, and four resistors is presented. Computer modeling results of the algorithm for calculate inductance was realized in the modern software LabView. Experimental results of realization of the equivalent of inductance are presented. The designed layout was applied for chaotic Chua's generator.

**Keywords:** Chaos, Equivalent Inductance, LabView, Chua's Generator.

# Dynamics of a Bertrand Duopoly Game with Differentiated Goods, Heterogeneous Expectations and Relative Profit Maximization

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In this article the authors investigate the dynamics of an oligopoly game in which, they consider a nonlinear Bertrand-type duopoly game with differentiated goods and heterogeneous expectations. In this study the case, where managers have a variety of attitudes toward relative performance that are indexed by their type is investigated. In this game they suppose a linear demand and cost functions. The game is modeled with a system of two difference equations. Existence and stability of equilibria of the system are studied. It is revealed that the models gives more complex, chaotic and unpredictable trajectories, as a consequence of change in the parameter k of speed of the player's adjustment, the parameter d of the horizontal product differentiation and the relative profit parameter  $\mu$ . The chaotic features are justified numerically via computing Lyapunov numbers and sensitive dependence on initial conditions.

**Keywords:** Bertrand duopoly game; Relative profit maximization; Discrete dynamical system; Nash equilibrium; Stability; Bifurcation diagrams; Lyapunov numbers; Strange attractors; Chaotic Behavior.

# On a Cournot Dynamic Game with Cost Uncertainty and Relative Profit Maximization

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In this paper, Cournot oligopoly model is examined with uncertain cost function. A random quadratic cost function is introduced in this model. The existence and uniqueness of the equilibrium are obtained. The asymptotic behavior of the equilibrium point is also investigated. Complete stability and bifurcation analysis are provided. The obtained theoretical results are verified by numerical simulations.

**Keywords:** Cournot duopoly game; Cost uncertainty; Relative profit maximization; Discrete dynamical system; Nash equilibrium; Stability; Bifurcation diagrams; Lyapunov numbers; Strange attractors; Chaotic Behavior.

Management of intrinsic localized modes in a driven nonlinear cyclic electrical transmission line

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Nonlinear electric transmission lines continue to play an important role in the understanding of the dynamics of intrinsic localized modes (ILMs). The development of a saturable nonlinearity with MOS capacitors in a cyclic transmission line has enabled the test to determine if transition points existed where the ILM could move freely. The evolution between large amplitude spatial modes (LSMs) and ILMs in a nonlinear cyclic electrical line with saturable capacitors also has been studied in some detail. The most dramatic feature is that by simply changing the driver frequency the spectrum can evolve continuously from an LSM pattern distributed around the ring to multiple ILMs localized on a few sites and visa versa. Through this novel nonlinear excitation and switching channel either energy balanced or unbalanced LSMs and ILMs may occur around the ring.

Keywords: Intrinsic Localized Mode, Spatial modes, electric cyclic lattice,

saturable nonlinearity.

## On the experimental excitation of modulation instability and Akhmediev breathers

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The smooth transition between stable, Talbot-effect-dominated and modulational unstable nonlinear optical wave propagation is described as a superposition of oscillating, growing and decaying eigenmodes of the common linearized theory of modulation instability. Results explain well the initial stage of experiments on modulation instability and breather excitation in a spatial-spatial platform. An increased accuracy of instability gain measurements and a more selective and well-directed breather excitation are demonstrated experimentally in a cascaded second-order nonlinear system.

**Keywords:** Modulation instability, Cascading, Akhmediev breathers.

## Increased mobility of discrete breather in lattices with odd inter-site and on-site anharmonic potentials

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The mobility of high-frequency discrete breathers in monatomic chains with on-site and inter-site potentials taking into account the nearest neighbors interactions is numerically investigated. It was found that the odd (cubic and fifth-order) interatomic anharmonic interaction strongly affects the mobility of breathers, sharply increasing the distance that it propagates without being trapped. The enlargement of propagating distance can exceed many thousands times. It was also found that the correctly chosen fifth anharmonicity leads to an inversion of stability between the bond-centered and site-centered breathers and to the lowradiative propagation of discrete breathers along the chain. According to our preliminary study, these conclusions hold also for discrete breathers in iron, copper, niobium and other simple metals. We also found a strong increase of mobility of discrete breathers by even on-site anharmonic potential; this increase may also exceed many thousands.

Keywords: Nonlinear dynamics, discrete breathers, intrinsic localized modes.

## **Homoclinic Orbits and Solitary Waves within the** Non-dissipative Lorenz Model and KdV Equation

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Recent studies using the classical Lorenz model and generalized Lorenz models present abundant features of both chaotic and oscillatory solutions that may change our view on the nature of weather as well as climate. In this study, the mathematical universality of solutions in different physical systems is presented. Specifically, the main goal is to reveal mathematical similarities for solutions of homoclinic orbits and solitary waves within a three-dimensional non-dissipative Lorenz model (3D-NLM), the Kortewegde Vries (KdV) equation, and the Nonlinear Schrodinger (NLS) equation. A homoclinic orbit for the X, Y, and Z state variables of the 3D-NLM connects the unstable and stable manifolds of a saddle point. The X and Z solutions for the homoclinic orbit can be expressed in terms of a hyperbolic secant function (sech) and a hyperbolic secant squared function (sech2), respectively. Interestingly, these two solutions have the same mathematical form as the solitary solutions for the KdV and NLS equations, respectively. After introducing new independent variables, the same second order ordinary differential equation (ODE) and solutions for the Z component and the KdV equation were obtained. Additionally, the ODE for the X component has the same form as the NLS for the solitary wave envelope. Finally, how a logistic equation, which is also known as the Lorenz error growth model, and an improved error growth model can be derived by simplifying the 3D-NLM is also discussed. Future work will compare the solutions of the 3D-NLM and KdV equation to understand the different physical role of nonlinearity in their solutions, and the solutions of the error growth model and the 3D- NLM, as well as other Lorenz models, in order to propose an improved error growth model to better represent linear and nonlinear error growth for both oscillatory and non-oscillatory solutions.

# Is Weather Chaotic? Coexistence of Chaos and Order within a Generalized Lorenz Model Country

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The pioneering study of Lorenz in 1963 and a follow-up presentation in 1972 changed our view on the predictability of weather by revealing the so-called butterfly effect, also known as chaos. Over 50 years since Lorenz's 1963 study, the statement of "weather is chaotic" has been well accepted. Such a view turns our attention from regularity associated with Laplace's view of determinism to irregularity associated with chaos. Here, a refined statement is suggested based on recent advances in highdimensional Lorenz models and real-world global models. In this study, we provide a report to: (1) Illustrate two kinds of attractor coexistence within Lorenz models (i.e., with the same model parameters but with different initial conditions). Each kind contains two of three attractors including point, chaotic, and periodic attractors corresponding to steadystate, chaotic, and limit cycle solutions, respectively. (2) Suggest that the entirety of weather possesses the dual nature of chaos and order associated with chaotic and non-chaotic processes, respectively. Specific weather systems may appear chaotic or non-chaotic within their finite lifetime. While chaotic systems contain a finite predictability, non-chaotic systems (e.g., dissipative processes) could have better predictability (e.g., up to their lifetime). The refined view on the dual nature of weather is neither too optimistic nor pessimistic as compared to the Laplacian view of deterministic unlimited predictability and the Lorenz view of deterministic chaos with finite predictability.

# Comparison between Q, $\Delta$ , $\lambda_2$ , $\lambda_2$ and Liutex methods of vortex-identification

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We are yet to get universally accepted methods of vortex-identification and it's physical reasoning and application. Since last three decades,

vortex-identification criteria like Q,  $\Delta$ ,  $\lambda_2$ ,  $\lambda ci$  etc. have been developed and remained popular in the field of vortex-identification. However, they are ambiguous and somewhat conflict with each other. So, to overcome this chaos situation in the vortex-identification field Dr. Chaoqun Liu, professor at University of Texas at Arlington, came up with the idea of Liutex method in 2018. Further, he gave the systematic definition of Liutex in scalar, vector and tensor form. In short, Liutex method gives the local rigid rotation of infinitesimal fluid element which clearly explains that direction of axis of rotation of the local fluid element is given by eigen vector associated with real eigenvalue of velocity gradient tensor and magnitude is given by the rotational strength of the fluid rotation on the plane perpendicular to the axis of rotation. In this paper, I am going to compare the Q,  $\Delta$ ,  $\lambda_2$ ,  $\lambda ci$  and Liutex methods with example and give the physical meaning behind why Liutex method has clear advantage over other methods with concrete examples.

### Chaos in Josephson nanostructures: short review

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We will discuss last year's results on chaos phenomena in different Josephson nanostructures including stack of intrinsic Josephson junctions in high temperature superconductors, shunted systems by inductance and capacitance, Josephson structures with ferromagnetic layers. We will present results on the effect of interjunction coupling in the coupled stack of junctions. The charging of superconducting layers, in a bias current interval corresponding to a Shapiro step subharmonic, due to the creation of a longitudinal plasma wave along the stack of junctions will be demonstrated. Recent results on synchronisation phenomena in shunted stack with different number of junctions will be discussed. Manifestation of chaos in the superconductor-ferromagnet-superconductor Josephson systems will be demonstrated.

**Keywords:** Chaotic modeling, Josephson nanostructures, Layered system, synchronization, SFS Josephson junctions.

## Overview of Scenarios of Transition to Chaos in **Nonideal Dynamic Systems**

#### **Aleksandr Shvets**

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A number of deterministic dynamic systems that are nonideal according to the Sommerfeld-Kononenko classification are considered. In particular, pendulum, hydrodynamic, and electroelastic systems with limited excitation are considered. The scenarios of transitions to chaos that are possible in the above systems are analyzed. We study both the transitions "regular attractor - chaotic attractor" and the transitions "chaotic attractor of one type — chaotic attractor of another type". In particular, the "chaos - hyperhaos" and "hyperhaos - hyperhaos" transitions are studied. Ten scenarios of transition to chaos are analyzed in detail. Some of the scenarios were widely known, while others are very unusual and are revealed only in nonideal dynamic systems.

Keywords: nonideal dynamic system, scenario of transition to chaos, chaotic attractor.

## Identification of Hidden and Rare Attractors in Some **Electroelastic Systems with Limited Excitation**

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Mathematical models of a deterministic dynamic system of the type "analog generator - piezoceramic transducer" are considered. A technique for searching for hidden and rare attractors of such a system is proposed. The transformations of hidden attractors into rare ones and vice versa are analyzed. The pairs of regular attractor - chaotic attractor are studied with respect to their qualifications in terms of "hidden" and "rare". Symmetry in the scenarios of the transition from regular attractors to chaotic attractors was revealed.

**Keywords:** hidden attractor, rare attractor, scenario of transition to chaos.

## Bifurcation and Chaos in a Discrete Prey-Predator Model

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In this work, a discrete predator-prey model with Holling type III functional response is investigated. It is shown that on varying a value of a bifurcation parameter, the system exhibits various type of bifurcation of codimension 1 including fold bifurcation, transcritical bifurcation, flip bifurcations and Neimark-Sacker bifurcation under certain conditions. Moreover, it is explored the existence of Bogdanov-Takens bifurcation of codimension 2 in the system and determined the expression for bifurcation curves. The extensive numerical simulation is performed to illustrate different type of bifurcation and chaos with respect to bifurcation parameter and other parameters in the system.

Keywords: Neimark-Sacker bifurcation, Codimension 1, Bogdanov-Takens bifurcation, Chaos.

## N-Rogue Waves in a New (2+1)-Dimensional Integrable **Boussinesq Model**

#### **Sudhir Singh**

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In this work, we have discussed the rogue wave solutions of a new (2+1)dimensional integrable Boussinesq model governing the evolution of high and steep gravity water waves. The evolution dynamics of obtained rogue waves along with the identification of their type, bright or dark type localized structures, and manipulation of their amplitude, depth, and width is also discussed. To construct rogue wave solutions, we used bilinear form and generalized polynomials. In particularly one, two and three order rogue wave solutions are obtained. The obtained results helps to demonstrate complete dynamics of rogue waves in higher dimension integrable systems and its various application over controlling mechanism of rogue waves in optical systems, atomic condensates, and deep water oceanic waves.

**Keywords:** Rogue Waves, Soliton Solutions, Integrable Systems, Bright/Dark Localized Structures, Generalized Polynomials, Rational Solutions.

# Analytical and Numerical Bifurcation Analysis of TLR4 Receptor Signaling Chaotic Dynamics

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We perform a one and two-parameter bifurcation analysis of a mechanistic model describing the dynamics of TLR4 signaling. We provide analytical results for fixed points and their stability with respect to the model parameters and the location of the Andronov-Hopf bifurcation points. We also perform a one-parameter numerical bifurcation analysis to trace the branches of oscillating solutions. By doing so we show that the dynamics include chaotic regimes emerging from a homoclinic bifurcation. Finally we perform a two-parameter numerical bifurcation of the Andronov-Hopf bifurcations and present the transition to Chaos via the Homoclinic Explosion.

**Keywords:** Bifurcation Analysis, Numerical Analysis, TLR4 Signaling, Homoclinic Explosion, Nonlinear Dynamics, Stability Analysis.

# Connecting Bernoulli and Schrödinger Equations and its Impact on Quantum-Mechanic Wave Function and Entanglement Problems

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An invariant model of Boltzmann statistical mechanics is applied to derive invariant Schrödinger equation of quantum mechanics from invariant Bernoulli equation of hydrodynamics. The results suggest new perspectives regarding quantum mechanics wave function and its collapse, stationary versus propagating wave functions, and wave-particle duality. The invariant hydrodynamic model also leads to the definition of generalized shock waves in "supersonic" flows at molecular-, electro-, and chromo-dynamic scales with (Mach, Lorentz, and Michelson) numbers exceeding unity. The invariant internal hydro-thermo-diffusive structure of such generalized "shock" waves are described.

### The evolution and breaking symmetry in the physics

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The nature of the deterministic symmetry breaking (DSB) in the classical and quantum dynamical systems is being studied. The nature of DSB can be revealed as a result of a complete description of the dynamics of systems only. A complete description implies taking into account the structure of the body, principle of dualism of symmetry and the using micro- and macro-descriptions of its dynamics. The need to take into account the body's structure is due to that the bodies full states and its dynamics is determined not only by macro-description, but also by micro-description. DSB in classical mechanics is associated with the mutual transformation of the system's motion energy and internal energy, provided that the sum of these energies is preserved. The mutual transformation of motion energy and internal energy occurs when a body moves in an inhomogeneous field of forces due to linking micro- and macro-variables. The transformation is determined by the bilinear terms of evolutionary nonlinearity.

It is shown, that in the approximation of local thermodynamic equilibrium, when a nonequilibrium system can be specified by a set of equilibrium structural particles SP's moving relative to each other, the model of thermodynamics potential function, that determines the DSB, has the form that coincides with the potential, which used for descrption of the spontaniously symmetry breaking for quantum systems [2]. This

coincidence indicates on the universality of the mechanism of symmetry breaking for quantum mechanics and classical mechanics.

The nature of the thermodynamic potential, which determine DSB in classical and quantum systems, is analyzed. A universal mechanism of DSB in classical and quantum systems is discussed.

**Keywords:** Entropy, irreversibility, determinism, evolution.

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# Problems of creating an evolutionary picture of the world

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The paper is devoted to study, how the physics of evolution allows developing an evolutionary picture of the world. Here we briefly examine the basic concepts of the world's picture and how based on the physics of evolution, these concepts can find development. For this purpose, the next questions will be analyzed: how physics of evolution leads to the conclusion about the infinite divisibility of matter; how nature solves the problem of the static state of matter, when motion is a way of existence of matter; how symmetry and its violation determine the evolution of matter; what are the principles of building "from simple to complex", how the process of evolution occurs according to the law of unity and struggle of opposites "chaos" and "order" etc. As a result, we show how taking into account the structure of matter in its dynamics leads to the possibility of describing evolutionary processes in natu! re. This means the possibility

of constructing a deterministic evolutionary picture of the world within the framework of the laws of physics

**Keywords:** Entropy, irreversibility, determinism, evolution.

"Chaos in Nonideal Dynamic Systems"

# Space-Timbre: a dynamical systems representation of foundational parameters in Electroacoustic Music

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Music can be seen as a phenomenological event that implies a whole series of relationships from dieerent realms ranging from acoustics, psychoacoustics and aesthetics and because of that is one of the most suitable arts for formal analysis through dynamical systems. For electroacoustic music there are two main characteristics that are developed through the compositional process and listening experience: timbre and space. Because of the very nature of the multichannel speaker format of this kind of music, the space has been approached theoretical and aesthetical by many important composers such as P. Henry, K. Stockhaussen, J. Chowning, F. Dohmont, F. Bayle, A. Vandergonne and M. Chion among others. Despite the fact that there are also many works that study timbre (as understood within the electroacoustic context) there are not too many theoretical works which propose an explicit and formal relationship between these two aspects. In Soria (2020) I proposed a theoretical model which stated as premise that space and timbre can be understood as a unique parameter: space-timbre and for developing that idea this parameter was stated precisely as a phenomenological object. The whole model (which was backened by a psycoacoustic-acousticperceptual framework) ended with a mathematical formalization of the space-timbre as a dynamical system. In this article this proposal is extended and detailed and the model is constructed through the basics from topological dynamics definitions. A first dynamical system is defined to represent the aural space (with a whole set of prior abstract objects that compose it). A second dynamical system is then associated to the spectromorphological space (the timbral characteristics of sound) and then by the universal property of the product of sets, the dynamical system

that defines the space-timbre is constructed. By this means a set of diderent definitions and results are proposed in order to achieve two main goals: a) develop a formal definition of the space-timbre with the dynamical systems foundations and b) create a unique and original mathematical representation of the space-timbre as a bridge between two realms, the electroacoustic music and the abstract mathematical language. In this sense this work contributes to the field of dynamical systems applied to music but with the novelty that it is focused to a new conception of musical parameter: the space-timbre.

**Keywords:** Dynamical Systems and Music, Systems Theory and Art, Electroacoustic Music and Math, Mathematical representations of Space and Timbre.

# The Universe multiphase meta-reduction: The Harmon (Mandala), continuum (Prana), discretization, formalization, knowledge, cognition, condensation and Absolute Nothing

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Universalization of the Universe concept allowed to deduce an alternative, more complete and consistent new cosmology, different from prevailing now the Big Bang hypothesis on the same factual basis. Obtained universal (chaotic / harmonic) cosmology is based on combinatoriallogical representations and fundamentally allows a qualitative derivation of universal properties in the framework of the axiomatic paradigm based on the hypothetical Universal Axiom (UA), the formulas of which are sufficiently substantiated. Intelligence is found to be the important universal concept immediately following from the UA that implements a conditioned reflex with a consistent concretization of hypotheses and minimizes the contradictions that arise. The only criterion of truth is exclusively the volume of the resulting hypothesis system, including facts. This gives rise to multivariance and hypothetical character knowledge of all kinds and justifies the applied research method, especially for conceptually remote areas such as cosmology. This research develops the above cosmology and substantiates a multiphase Universe, which consistently passes through different phase states with a reduction in

propertiesion that allows effective cognition and harmonization up to achieving the target state of Harmon followed by complete self-destruction, in contrast to the prevailing theories of continuous development of the Universe. The study of phases opens up new important Universe's properties unknown to modern science, but well consistent with it. All the used meta-concepts are sufficiently formalized, however, it is difficult to understand in terms of our surrounding world, but fundamentally necessary for its existence. The meta-results obtained have the widest direct or indirect application, without which any serious research and development becomes incomplete and misunderstood.

**Keywords:** The Universe, Cosmology, Universal Theory, Universalization, Chaos, Harmony.

# Comparison of Vortex Identification Methods for Corner Separation flow in a Compressor Cascade

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The three-dimensional (3D) corner separation, which is an inherent flow structure within the compressors, has great effect on compressor performance, such as passage blockage, limiting static pressure rise, a considerable total pressure loss, reduction in compressor efficiency, and eventually stall and surge especially for highly loaded compressor. Hence, the flow mechanism should be further investigated. Vortex identification criterion is an important tool analyzing this kind of complex flow.

In the current study, the delayed detached eddy simulation (DDES) is firstly conducted in a highly loaded compressor cascade at the Mach number of 0.59. Then the different vortex identification methods, such as vorticity, pressure,  $\tilde{A}, Q, -\lambda_2, \lambda_{cv}^2, \Omega$ , local trace criterion ( $LT_{cri}$ ) and Liutex criteria have been used to analyze the 3D corner separation based on the DDES results. Detailed results for S1 and S3 stream surfaces are compared. The results show that different criterion has almost the same result when the vortex is concentrated and stable. However, they behave different as the corner separation becomes unstable. The Liutex criterion, which is directly related to fluid rigid rotation, can filter out the shear layer influence close to the wall. The vortex vector can also be used to show

the vortex structure by Liutex vector lines. The  $(LT_{cri})$  could indicate different swirling patterns for the 3D corner separation flow.

**Keywords:** Vortex indentification, Corner separation, Flow visualisation, Compressor.

# Study of Vortex Structure in a Linear Compressor Cascade Using Liutex and Local Trace Criterion

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Different kinds of vortical structures, such as horseshoe vortex, passage vortex, wake shedding vortex, and corner vortex, exist in compressors. These complex vertical flows play an important role in compressor performance. Thus, it is of great meaning to gain deeper insight into vortex structure through advanced analysis method. In the current study, the delayed detached eddy simulation (DDES) is conducted in a highly loaded compressor cascade at the Mach number of 0.59. The Liutex method and local trace criterion (LTcri) are employed to analyze vortical flow structures based on the DDES results.

Contours of different stream surfaces marked by Liutex and LTcri are analyzed. The Liutex method effectively extracts the rigid region, and excludes the shearing contamination. The LTcri indicates different swirling patterns for the cross section of vortex identified. Detailed analysis is made using these two vortex identification methods in visualizing the vortex structures in typical stream surfaces. The vortex core is extracted by Liutex core method and the evolution of its swirling pattern is analyzed by the LTcri based elliptical region method. The probability density distributions are analyzed for the Liutex core region in the division of projected elliptical region. Corrections of these two vortex identifications are also discussed. A self-correlation threshold is explored for the Liutex to extract the region that projected in the LTcri based elliptical region bound.

**Keywords:** Vortex indentification, Vortex structure, Flow visualisation, Compressor.

## The ac driven Frenkel-Kontorova model: from Shapiro steps to chaos

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The appearance of devil's staircase and chaos have been studied in the dc+ac driven Frenkel-Kontorova model. In the overdamped limit, the devil's staircase structure arising from the complete mode locking of an entirely nonchaotic system was observed. Even though no chaos was found, a hierarchical ordering of the Shapiro steps was made possible through the use of a previously introduced continued fraction formula. When the inertial term is included, unlike in the overdamped case, the increase of mass led to the appearance of the whole series of subharmonic steps in the staircase of the average velocity as a function of average driving force in any commensurate structure. At certain values of parameters, the subharmonic steps became separated by chaotic windows while the whole structure retained scaling similar to the original staircase.

Keywords: Frenkel-Kontorova model, Shapiro steps, chaos.

#### Discrete Rogue Waves in a non-Hermitian Waveguides

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SHMYO, Kırklareli University, Kırklareli, Turkey Department of Physics, Eskisehir Technical University, Eskisehir, Turkey Extremely large amplitude waves, called as 'rogue waves', 'freak waves', 'extreme waves', 'giant waves', 'huge waves', 'killer waves', etc. , have attracted the great attention of researchers because of their interesting feature [1]. They suddenly appear on the sea surface and abruptly disappear [2]. They are at least two times higher than the rest of the sea [3]. They can be observed not only on the sea surface but also in other physical fields [4]–[7]. In this study, we show that rogue wave evolution changes with non-Hermitian waveguides. The hopping amplitudes in the magnitude of the forward and backward direction indicate the site where rogue waves occur.

**Keywords:** Rogue waves, non-Hermitian system.

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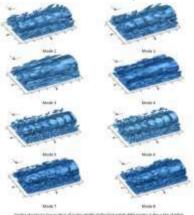
# Liutex and Proper Orthogonal Decomposition for Vortex Structure in the Wake of Micro Vortex Generator

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A so-called new Omega vortex identification criterion was proposed by Liu et al. In 2016, which has many advantages as mentioned in previous study. However, all the existing vortex identification methods are just a scalar and they are contaminated by shear. At the first time in the history, a new vortex vector with both the local rotation axis and rotation strength - called Liutex (with no shear) was introduced by Liu et al in 2018. In this study, we load data directly from Liutex vector  $(L_x, L_y, L_z)$  instead of velocity (u, v, w). Besides, proper orthogonal decomposition (POD) method is applied to analyze the coherent structure of first eighth POD modes in the wake of micro vortex generator at Ma = 2.5 and  $Re_\theta = 5760$  by using Liutex. Pairing of POD modes is clearly found, which is the sign of Kevin-Helmholtz instability. The observation of the mode paring strongly supports the new mechanism of MVG for reduction of flow separation, which is the spanwise vortex ring generation by KH instability.

**Keywords:** Liutex, proper orthogonal decomposition, micro vortex generator, Kevin-Helmholtz instability.



Switching Frequency Bifurcations in a LED Boost Driver

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Under specific conditions LED driver circuits can be as susceptible to chaotic bifurcations, as conventional boost converters have proven to be. A significant relationship between the switching frequency of the boost converter's transistor and the circuit's nonlinear behaviour is shown. In order to examine such transistor switching frequency effects, an openloop configuration is employed, since a feedback control system would obscure these particular nonlinearities. We have devised a theoretical method to predict the unstable frequency regions based on certain dependence equations. We have witnessed that particular nonlinear parameters influence the circuit's behaviour, such as the reverserecovery time of the boost diode, as well as the collective effect of the inductance and the diode's junction capacitance. The dependence equations prove a correlation between these inherent nonlinearities and the switching frequency of the boost transistor. Period doublings and transitions to chaos occur for several regions of the examined switching frequency range. The theoretical method of our numerical analysis is based on the periodicity of certain voltage waveform peaks, probed at key points on the converter. The LED boost driver displays a wealth of nonlinear phenomena and detrimental effects on its brightness levels throughout the nonlinear frequency regions.

**Keywords:** LED boost converter, chaotic oscillations, nonlinear dynamics, bifurcations, lighting circuits.

# Existence of Solutions for a System of Fractional Boundary Value Problems

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We study the existence of solutions for a system of Riemann-Liouville fractional differential equations with nonlinearities dependent of fractional integrals, supplemented with uncoupled nonlocal boundary conditions which contain various fractional derivatives and Riemann-Stieltjes integrals. In the proof of our main results we use some theorems from the fixed point theory.

**Keywords:** Systems of Riemann-Liouville fractional differential equations, Fractional integrals, Nonlocal boundary conditions, Existence of solutions.

# Hysteresis loops, dynamical systems and magneto optics

#### Adriana Pedrosa Biscaia Tufaile, Alberto Tufaile

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The interest in hysteresis and magnetism is shared by scientists with an impressive variety of backgrounds, such as mechanics, thermodynamics, electromagnetism, catastrophe theory, mathematics and dynamical systems, because hysteresis loop is a concept at the core of non-linear systems in which the dependence of the evolution of states of these systems are related with their history. In this work we present a connection between dynamical systems and hysteresis loops and after that, we present some interesting hysteresis loops obtained in some magnetooptical systems using the Transverse Magneto-Optical Kerr effect (TMOKE), such as thin films of amorphous alloys with weak gadoliniumcobalt and strong holmium-cobalt random local anisotropy by thermal behavior. We compare some hysteresis loops of ferromagnetic, paramagnetic, superparamagnetic and sperimagnetic systems. The comparison between magnetic and magneto-optical hysteresis loops for gadolinium-cobalt has shown that the spin-reorientation phase transition occurs from a colinear phase to an opposite collinear phase. In this case, the transition magnetic field grows exponentially when the temperature increases.

**Keywords:** Hysteresis loop, Sperimagnetism, Transverse Magneto-optical Kerr effect.

## Halo dynamics: from rainbows to black holes

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Light halo formation is one of the most ubiquitous visible phenomena in nature, and we can observe these halos from rainbows and soap bubbles [1] in everyday life to the bizarre behavior of light in the presence of huge gravitational fields, such as black holes. In order to understand all these systems in a practical way, we are proposing this study of optical systems based in dynamical systems. First, we explore the existence of multiple rainbows and its connection to the light ray dynamics of a cylindrical lens. After that we study the case of a lens with the shape of a foot glass, commonly used to simulate the gravity field effects described by Einstein to test the theory of relativity. Using a cylindrical shell, we have obtained the pattern of an involute for a circle, like the one studied by Huygens when he was studying clocks without pendula for use on ships at sea. We also explore some Möbius transformations of spheres and pseudospheres in connection with image formation, presenting some experiments reproducing these concepts, such as cusp catastrophe and cartesian rays, arc formation, the spiralbow [2, 3] and periodic behavior of light rays in a cylinder, twin rainbows, solar halos, bifurcations of light path in the presence of massive objects and Einstein rings, combination of gravitational lenses forming multiple images and arcs. This method gives a intuitive perspective of many systems involving halos and similar

Keywords: Halo, Parhelic Circle, cusp catastrophe, hyperbolic prism, Einstein rings.

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## **Investigating dynamical systems using Optic-Fluidics**

### Alberto Tufaile<sup>1</sup>, Michal Snyder<sup>2</sup>, Timm A. Vanderelli<sup>3</sup>, Adriana Pedrosa Biscaia Tufaile<sup>1</sup>

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Since Faraday, the concept of magnetism is strongly connected with the creation of the concept of field. In this work we present experimental results and simulations of dynamical systems using magneto-optics [1, 2, 3], as shown in Fig. 1. This light pattern is obtained by the observation of a thin film of ferrofluid in the presence of a magnetic field. We can compare the light patterns with trajectories of phase space in dynamical systems. The simulations of this system were done using the software Pic2Mag [4].



Fig. 1 – Ferromirror image obtained usigng mageneto-optical device.

**Keywords:** Non-linear system analysis, Ferrofluids, dynamical systems, magnetism.

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## Vibrational Resonance in inhomogeneous and spacedependent nonlinear damped systems

B.I. Usama, S. Morfu, M. Rossé, P. Marquié, J.M. Bilbault ImViA EA 7535, Univ. Bourgogne Franche-Comté, Dijon, France

The properties of nonlinear systems have attracted a considerable interest these past years since they can be used to develop bio-inspired applications ranging from signal detection to image processing. Among these nonlinear signatures is Vibrational Resonance (V.R.) which expresses the ability of a nonlinear system to take benefit of a high frequency perturbation in order to enhance its response to a weak low frequency excitation [1,2]. Since its introduction in mechanical systems [1] and electronic devices [2], this effect has been widely reported in various systems and for different applications [3]. Indeed, V.R can be used to detect subthreshold signals [4] or to enhance the perception of subthreshold images [5]. Among the systems which exhibits V.R., we can cite bio-inspired circuits [6,7], networks of coupled electronic neurons [8], as well as inhomogeneous and space-dependent nonlinear damped systems [9]. These last mechanical systems constitute the subject of our communication which adresses more specifically the conditions of existence of V.R. The system under numerical investigation models a particle of mass m submitted to a multistable potential and a space dependent nonlinear damping. After showing that multiple resonances can be controlled by the system parameters, we show that the particle mass can allow or hinder V.R.. Indeed, V.R. ceases to exist for particles exceeding a critical mass [10].

**Keywords:** Vibrational Resonance, nonlinear dynamics.

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# Ways to Accelerate Nanotechnologies Implementation in the Health Care System

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Commercialization of nanotechnologies in Russian Health Care System RHCS requires detail analysis of all obstacles. Here we discuss 3 problems: Prof. Ilizarov's apparatus; 'Perftorun', known as 'blue blood' therapy of Russian prof. Beloyartsev; 'Litar' and artificial bone technology of Prof. Krasnov, used to replace bones defects. We look into challenges of Russian nanotechnology clusters and education. Prof. coauthor of our communication, has his own rich experience in implementing new nanotechnologies, used to treat injured military personnel in Russian armed conflicts, such as Chechen war. Russian technology innovations require 30 - 40 years for commercialization, 5 - 10 years in USA Substantial investment capital significant stringent requirements and high chance of technology failure in preclinical or clinical trials hinder this development in developed countries. Stringent regulatory approval state process further increases time and cost its moving to the market. Patent protection is often key strategy to attract multimillion investment required for early stage transition of medical technology into commercial product. Our proposal is to enhance commercial translation of 3 above mentioned organizations. Then enable treatment of severe bone fractures and injuries, here patient's own tissues cannot be employed. These technologies were validated through surgical procedure performed in military. RHCS authorities are evaluated based on how efficiently they conduct certification and regulatory approval of new medical technologies.

#### **Application of Special Nanomaterials in Medicine**

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Extensive research of biomaterials at nanoscale has led to development of novel medical technologies including prosthetic devices and new surgical material and methods. In many cases, however, biomechanical properties and health impacts of nanomaterials are poorly understood. The FDA has even established a separate group within the Agency to develop better knowledge of interactions of nanomaterials with biological systems, and to assess the adequacy of testing approaches for evaluating safety, effectiveness, and quality of products containing nanomaterials. Here we present three key innovations used in treatment of severe bone injuries among veterans and athletes: (i) Prof. Ilizarov's apparatus, (ii) 'Perftorun', known as 'blue blood' therapy discovered by Prof. Beloyartsev in Russia, (iii) 'Litar', an artificial bone technology invented by Prof. Krasnov that is used to replace bones defects. Prof. Petrov, a coauthor of this paper, has extensive experience in implementation of novel technologies for health protection and safety including the use of the above mentioned technologies. Further we present our analysis of key challenges that hinder commercialization of new biomedical technologies and limit the use of such technologies in human patients. By way of example, we will illustrate how suboptimal regulatory approval process for new biomedical devices can substantially increase the time and cost of technology translation from bench to bedside and will discuss the importance of technology's patent protection in attracting private investment required for commercialization of biomedical technologies. Lastly, using the three above-mentioned innovations as our case studies. we will suggest approaches for improving the outcomes of biomedical technology translation. Nanoworld is very amazing creation and isn't a simple significant reduction in surrounding objects, but lives and functions according to its own laws, according to which our civilization was conceived by the Total World Creator. We are trying to get as close as possible to Creator's intentions, including through correct choice of medical materials and architecture of their construction in muscles and bone tissues for speedy implementation and adaptation into human organism as result of regular and random numerous factors. Commercialization of new nanotechnologies in Russian Health Care System (RHCS) has some serious obstacles. Here we discuss 3 above mentioned problems and creation of new nanotechnology clusters and education to accelerate implementation of its in RHCS. Prof. Petrov,

coauthor of our communication, has his own rich experience in implementing new nanotechnologies, used to treat injured military personnel in Russian armed conflicts, such as Chechen war. Russian technology innovations require 30 - 40 years for commercialization, 5 - 10 vears in USA. Substantial investment capital significant stringent requirements and high chance of technology failure in preclinical or clinical trials hinder this development in developed countries. Stringent regulatory approval state process further increases time and cost its moving to the market. Patent protection is often key strategy to attract multimillion investment required for early stage transition of medical technology into commercial product. Our proposal is to enhance commercial translation of 3 above mentioned organizations. Then enable treatment of severe bone fractures and injuries, where patient's own tissues cannot be employed. These technologies were validated through surgical procedure performed in military. RHCS authorities are evaluated based on how efficiently they conduct certification and regulatory approval of new medical technologies. Keywords: Biological systems, Biomedical technologies, Medicine nanomaterials, prosthetic devices.

### Statistic Methods for Assessments of Risks and **Damages at Nuclear Power Plants**

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Today NPP using for energy production is constantly increasing in the world with the growth of different threats, for example, caused by the natural and manmade catastrophes and directed terrorist attacks [1]. It is necessary to make possible correct assessments of corresponding risks levels in common case and for single separated NPP from moment of NPP projecting, building and especially during its exploitation. It demands to provide exclusive attention and special conditionsunder realization of complex integrated emergency NPP management [1].

We use our universal formula for assessment of the total vector of limited losses under NPP exploitation. The main problem is assessment of loss probability matrix elements [2]. If representative statistic data, obtained for long NPP exploitation period, are present, then some of its elements may be assessed by statistic methods. We have predicted the irradiation doses and corresponded risks for population under implementation of Russian Federal Program:" Development of Russian atomic energy industrial complex on 2007-2020 years at 10 Russian NPP that operated during some last decades [3]. But early some types of NPP disasters were absent at researched NPP. Using of classic methods of expertise assessments is not correct in this case. Some needed data may be obtained from primary virtual computer tests of concrete NPP with imitation of possible disasters. It allows to assess risk values and also to plan the actions for NPP operators and special services under serious NPP disasters or may be to prevent them at all. The following important aspects and problems are under consideration in our communication: 1. The NPP researchers by our statistic methods [4]; 2. Assessments of Risks and Possible Ecological and Economic Damages from Large-Scale Natural and Manmade Catastrophes in Ecology-Hazard Regions of Central Asia and the Caucasus [2, 3]. 3. Some particularizes, connected with the future development of nuclear energy industrial complex in Kazakhstan regions of Ust - Kamenogorsk city and NPP future construction near Lake Balshash and Kurchatov city are also presented in our communication.

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# Integrated Emergency Management and Risks for Mass Casualty Emergencies

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Today it is observed the intense growth of various global wide scale threats to civilization, such as natural and manmade catastrophes, ecological imbalance, global climate change, numerous hazards pollutions of large territories and directed terrorist attacks, resulted to huge damages and mass casualty emergencies. It stimulates modernization of traditional methods and development of new ones for its researching, prediction and prevention with maximum possible decreasing of their negative consequences. The global issue of safety provision for the humankind is the most actual and requires an immediate decision. The humankind has faced the majority of treats at the first time, therefore, there are no analogues and means to be used for their solving. Catastrophe risks have increased so much, that it becomes evident that none of the states is able to manage them independently. Join efforts of all world community are necessary for the substantial development of our civilization. We point out the main following obstacles for this realization: (1) Every country has its own political, economical and demography particularities, that greatly reflect on behavior of emergency actions in critical situations: (2) Serious international frictions between different courtiers often take place especially at its transboundary territories; (3) Many vast territories are localized in zones of so called "frozen" or operated international and region conflicts, that in addition promote

realization of different directed terrorist acts; (4)Contradictions interests between of all community (including state governments) and the local private and international industrial companies, that realize its activity in separate country; (5) Insufficient level of population's safety culture; (6) Using of traditional and classical methods for disaster's investigations is often non effective and has failures. The brittle equilibrium between nature and human civilization has been broken now. Our Earth replies and revenges us. Sometimes it is impossible to predict natural and manmade disasters, catastrophes and terrorist acts with using of explosives. It demands very high organizational functions of all special emergency services - informational, fire, evacuation, searching of casualties with immediate realization of medical help. International Political Thematic Programs and Methods have been created. Its purposes and functions are considered here. UNO Global International Strategy includes the unite of the total directed efforts of all scientific, educational, economical, social, state and private organizations and emergency services all over the world. The authors have their own experience and methods in this direction. Some special examples are under consideration and analyzed. We pay the special attention to educate of safety culture at population that increases its alert to risk accidents and behavior. The special events and meetings that realize by three forms Scientific Program of NATO (NATO ASI, ARW, ATC), promote development of safety culture. It is very important to remember that the wide scale catastrophes have not any boundaries and anv political and economical frictions between some states are not the reasons for the implementation of the struggle against them. The total emergency recommendations and actions have to be improved to eliminate and software of negative disaster's responses on population and environment. We present some examples of realization of Integrated Emergency Management and using of special methods and techniques in the most critical situations that have taken place in different countries in 21 century.

**Keywords:** management, risk, threats, catastrophes, terrorist attacks, emergency services.

Inversive generators of second order

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Inversive congruential method for generating the uniform pseudorandom numbers is a particulary attractive alternative to linear congruential generators, which show many undesirable regularities. In present paper, we investigate the equidistribution of sequences produced by inversive congruential generator of second order by using the discrepancy bounds of such sequences of pseudorandom numbers (PRN's). Also there are obtained the estimates of special exponential sums of these sequences. **Keywords:** inversive generator, exponential sum, discrepancy

# Predicting the dynamics of nonlinear instabilities: disruptions in Tokamaks

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The disruptive instability in tokamaks is characterised by a fast loss of the plasma confinement and a rapid quench of the plasma current. The consequences of disruptions can be dangerous for both the plasma facing components and the vacuum vessel. Therefore, the prediction and recognition of disruptive behaviours is crucial in the operation of tokamaks to apply either avoidance or mitigation actions. However, so far, there are no physics models able to completely explain the origin and diversity of disruption types, which makes impossible the prediction of disruptions with success rates close to 100% from physics reasoning. To overcome the lack of physics models, machine learning methods have been used to develop data-driven models. These models have shown high success rates (> 95%) and low false alarm rates (< 5%). This contribution

summarises the evolution of disruption predictors during the last years: from predictors that require a lot of training data to adaptive predictors from scratch, from multi-dimensional spaces with very complex separation frontiers (from the mathematical point of view) to two-dimensional spaces with a linear frontier, from extremely complex predictors of the time to the disruption to one-dimensional models based on Weibull distributions. Results with JET data will be provided.

### Chaos and Global Indeterminacy in an Environmental Economic Growth Model

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In this paper, we consider an environmental economic growth model (Wirl, 2004). The steady state analysis is conducted. The dynamic models with two renewable resources possess two state variables. The system exhibits multiple steady states and gives rise to some ecological management problems. We proved the existence of a saddle focus equilibrium. In order to find cycles, complex dynamics and indeterminacy results, we apply global bifurcation instruments to our system. We focus on a parameter region of local determinacy. We show the possibility of global indeterminacy and chaos in its subset (Mattana, P., Nishimura, K., Shigoka, T., 2009, Antoci, A., Galeotti, M., Russu, P., 2014). Our route to chaos exploits the existence of a homoclinic orbit to the saddle-focus equilibrium. The dynamics near these homoclinic orbits has been discovered and investigated by using the Shilnikov Theorem (see Bella G., Mattana P., Venturi B. 2017). This involves hyperbolic horseshoes close to the homoclinic orbit, but possibly also periodic attractors and strange attractors. It might be impossible to characterize the system for a full set of parameter spaces, and the boundary of a chaotic region. The economic implications are finally motivated and discussed and numerical analysis is performed.

**Keywords:** Environmental Economic Growth, Homoclinic orbit, Shilnikov Theorem.

# Growth, Sustainability and Ggreen Poverty Traps In a **Simple Integrated Model**

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In this paper we present a simple environmental economic growth model. The accumulations of capital and pollution are connected by reciprocal feedbacks. Pollution is an undesirable but inevitable by-product of production whose efficiency is hindered by pollution. The nexus between pollution and production is embodied in a damage function. The dynamics of the system is enriched by the specific technology adopted here, that is an S-shaped production function. An analysis of the stability of the multiple equilibria arising from the model underlines the importance of a meditated choice of the relevant policy parameters. A Hopf bifurcation preludes the emergence of a green poverty: all the economies with initial conditions outside the region circled by the Hopf bifurcation are doomed to collapse. A global numerical analysis corroborates our results and provides more insights on the existence of green poverty traps.

**Keywords:** Economic Growth, Pollution, Poverty Traps.

# **Chaotic Mixing Experiments at High Temperature: Unravelling a Large Magmatic Province**

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The Paraná-Etendeka Magmatic Province (PEMP) is the second largest outpour of magma on the Earth surface during lower Cretaceous times (~133 Ma). Basalts (~50% SiO2, both high-Ti and low-Ti members) predominate over an estimated volume of 7 • 105 km3. However, ca. 2.5% of the volcanic products, are chemically more evolved (>63% SiO<sub>2</sub>). Their genesis is still under debate. This work aims a first attempt to experimentally reproduce the impact of underplating basaltic melt into a pre-existing continental crust. Mixing dynamics is thought to greatly influence the formation/contamination conditions of the high-Ti acid member (Chapecó-type). We used a chaotic mixing protocol (Journal Bearing System) at 1,350oC and following end-members: ;  $\eta_{1350} = 8.78 \text{ Pa.s}$ ;  $\rho_{1350} = 2.469 \text{ g/cm}^3$ ) and LMC-027 granite (syenogranite from Capão Bonito Stock;  $\eta_{1350}$ =  $1.22 \cdot 10^5$  Pa.s;  $\rho_{1350} = 2.292$  g/cm<sup>3</sup>). Homogenized glasses from the basalt and the granitic basement were used in a proportion 80/20, respectively. The experiment was performed during 212 min in total, i.e., two periods of: (i) two clockwise rotations of outer cylinder (35 min); (ii) six anticlockwise rotations of inner cylinder (18 min). The independent and nonsimultaneous movements of the two cylinders guaranteed the chaotic flow. The obtained mixed glass was cut in slices of 3 mm perpendicular to the rotation axes and two of the sections reproduced Poincaré patterns, which are theoretically expected to be resulted for this kind of dynamic mixing. With the development of stretching and folding processes, chaotic trajectories are distributed off-centered as lamellar lens-like structures in the mixed system. Vortex structures are comparable to those produced by mixing Fe-free silicate melts, however with much higher fractal dimension (D<sub>f</sub>). These sections were preliminarily analyzed for the changes in morphology by comparing the D<sub>f</sub> using binary images (ImageJ software). Obtained  $D_{f}$ 's (~ 1.78) are close to those from similar experiments with natural melts, although widespread orbicular structures found along all basaltic morphological domains are thought to enhance the complexity. Further experiments varying the granitic end-member are planned. Raman, microprobe and LA-ICP-MS investigations will be performed to compare chemical behavior of major/minor oxides and trace elements. Furthermore, numerical simulations will follow.

**Keywords:** Magma mixing, chaotic dynamics, journal bearing system at high T, Paraná-Etendeka Magmatic Province, Chapecó-acidic type.

Unveiling the Characteristics of Stick-slip Oscillations in a Piecewise Smooth Rotor/stator Rubbing System

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Self-excited dry friction backward whirl, or shortly denoted as DFBW, is induced due to the dry friction effect at the contact point and the most destructive response in rotor/stator rubbing systems. Stick-slip oscillations are the typical dry friction induced motion. This letter will thus investigate the characteristics of stick-slip oscillations exhibited in DFBW from the point of view of non-smooth sliding bifurcations. Three types of sliding regions are identified on the curved hypersurface of the switching manifold, which is defined by the zero relative velocity on the rubbing point in a four-dimensional piecewise smooth rotor/stator rubbing system. The interplay of the system parameters on the stick-slip transitions in DFBW is revealed and coincides with the phenomena observed experimentally. Furthermore, a new way to analytically derive the existence condition of DFBW is presented on basis of the relation between the existence boundaries of DFBW and the characters of sliding motions on it, which are in good agreement with the existing formula in the literature. The work provides a deeper insight into the response characteristics in DFBW of rotor/stator rubbing systems.

Keywords: piecewise smooth systems, stick-slip oscillations, rotor/stator rubbing, dry friction backward whirl, existence condition.

#### Lagrangian Liutex

#### **Yiqian Wang**

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Liutex vector is a recently introduced quantity which can precisely extract the rotational part of the fluid motion. Being an Eulerian method, the Liutex vector has the advantage of ease to use, systematic and providing velocity gradient tensor decomposition. In this paper, we introduce the Lagrangian Liutex which measures the rigid motion in the considered time period. Besides being a practical Lagrangian vortex identification method, the Lagrangian Liutex can also be viewed as a coherence measurement of the Liutex vector, which will be illustrated by several analytical examples.

**Keywords:** Liutex vector, Lagrangian vortex identification method.

### Liutex application in the cavitating flows around a threedimensional bullet

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The objectives of this paper are to (1) provide a better insight into the dynamics and structures of the transient cavity, (2) improve the understanding of the interaction between unsteady cavitating flow and vortex dynamics with different vortex identification methods. Numerical simulations of the unsteady cavitating flows are conducted around a three-dimensional bullet by using a modified Partially-Averaged Navier-Stokes method via the open-source code, OpenFOAM. The results are compared with that predicted by the standard k-\(\perc \text{ turbulence model, indicating that the modified PANS model is reliable to predict the unsteady cavitation patterns, including the cavity initiation, growth and subsequent shedding. In order to clarify the mechanism of the cavitation-vortex interaction, different vortex identification methods are introduced to visualize the vortex structures. Finally, further analysis of the vortex dynamics is illustrated by using the vorticity transport equation, the results show strong correlation between the cavity and the vorticity structure.

Keywords: PANS, OpenFOAM, vortex identification, Liutex.

# Observation of propagation of nonlinear localized oscillations in a mass-spring chain with excitation and attenuation ends

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Mobile type of nonlinear localized oscillations have been excited by making mechanical experiments using a mass-spring chain which emulates the Fermi-Pasta-Ulam (FPU) one of □□type [Phys. Lett. A 382 (2018) 1957-1961]. Letting the weight at one end of the chain driven sinusoidally with high frequency and large amplitude, localized oscillations

have been excited intermittently near the end and propagated down the chain one after another at a constant speed. Because of the finite length of the chain, the localized oscillations traveling and reflected at the other end immediately interact and make the oscillation mode of the chain complicated. To pick up and observe one-direction propagation of the localized oscillations, we set an apparatus near the driving end to control the approach of excited oscillations into the chain and provide attenuation effects at the other end to suppress the reflection. In this paper we consider the relation of the speed of the propagation tothe driving frequency and amplitude.

**Keywords:** Intrinsic localized mode (ILM), Discrete breather (DB), discrete nonlinear system, Fermi-Pasta-Ulam (FPU) chain, piecewise linear spring, mobile ILM.

# Session title: Mathematical analysis of models arising from biology and physics

#### Liutex In the vortex statistics of 2D turbulent system

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Liutex is applied for the vortex statistics in two-dimensional (2D) turbulence based on Immersed Boundary Lattice Boltzmann Method (IB-LBM). There are three different turbulent systems investigated: (1) a circular cylinder placed near a plane wall in 2D channel; (2) arrays of cylinders placed both horizontally and vertically in 2D channel; (3) the cylinders placed in Rayleigh-Bénard flow. The results of vortex statistics for all cases show that the algebraic number density  $n(A){\sim}A^{-2}$ , where A is vortex area. It is found that Liutex has more advantages than  $\lambda_2$ , Q and  $\Omega$  in 2D vortex statistics.

**Keywords:** 2D turbulence, Liutex, vortex statistics.

Numerical Study on influence of vortex structure of jet in crossflow in axisymmetric transonic nozzle

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In order to study the influence of vortex structure of jet in crossflow in transonic nozzle, a typical transonic nozzle was simulated by Large Eddy Simulation and Liutex was adopted to identify vortex structure. The typical characteristics and vortex structure of transonic nozzle are analyzed in detail. A new vortex structure was found downstream of the jet at a pressure ratio above 1.5. Compared with jet in supersonic crossflow, it's not quite the same vortex structure in jet in in crossflow in axisymmetric transonic nozzle. The structure, development and evolution of the vortex are analyzed in detail under different pressure ratio and injection angles. This study is helpful to explain the physical mechanism of aerodynamic throat regulation.

**Keywords:** Jet in crossflow, Transonic, Liutex, Vortex structure, Simulation, Nozzle.

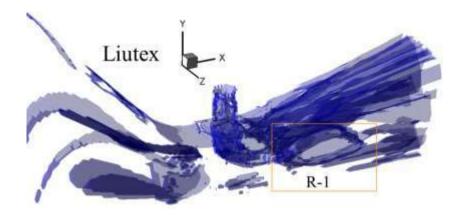


Figure 1 The vortex structure in transonic nozzle identified by Liutex=0.52

Figure 2 Vortex structure in reverse injection

# No vortex in flows with straight streamlines - Some comments on real Schur forms of velocity gradient ∇v

#### Xiangyang Xu, Wennan Zou

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It is widely recognized that the flows with straight streamlines have no vortex. Various vortex identification methods based on velocity gradient, such as Q criterion,  $\Delta$  criterion,  $\lambda$ 2 criterion,  $\lambda$ ci criterion,  $\Omega$  criterion, etc. were developed to separate tube-like vorticity structures from the platelike ones. However, underlying all these approaches, including the vorticity method itself, is the deformation analysis decomposing velocity gradient into strain rate and rotation rate. In recent years, a new trend to accept the simple shear as an independent mode of fluid flow becomes obvious. To excavate vortex features from the velocity gradient with complex eigenvalues results in an intensive study on the real Schur forms of velocity gradient. Zhou et al. deduced the local streamline pattern, He et al. proposed the concept of canonical rotation, and Liu and his coworkers further explored the characteristic information of the velocity gradient, presented the so-called Liutex blueprint for vortex identification. In this paper, we point out that the canonical rotation in both He's and Liu's studies is derived from the real Schur form of the left eigen problem of velocity gradient. We supplement the real Schur form of the right eigen problem of  $\nabla v$ , and give the tensor decomposition representations and geometric meanings of all of them. We further present the relations between two representations, and especially about the parameters characterizing the vortex. Using the DNS velocity fields, the connections and difrences between the two real Shure forms are illustrated.

**Keywords:** Velocity gradient, Eigen problem, Real Schur form, Liutex, Vortex identification.

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### Time Dependent Stabilization of a Hamiltonian System

#### Asher Yahalom, Natalia Puzanov

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In this paper we consider the unstable chaotic attractor of a Hamiltonian system with Toda lattice potential and stabilize it by an integral form control. In order to obtain stability results, we use a control function in an integral form:  $u(t)=\inf_{0}^{t}k(t,s)X(s)ds$ , in which all the back story of the process  $X_{(t)}$  is taken into consideration. Using the exponential kernel  $k(t,s) = e^{-t}(t-s)$ , we replace the study of integro-differential system of order u(t-s), we replace the study of integro-differential system of order u(t-s), with an analysis of u(t-s), order system of ordinary differential equations (without integrals). Numerical solution of the resulting system leads to the asymptotically stabilization of the unstable fixed point.

# **Correlation Analysis between Vorticity and Liutex** (Vortex)

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Influenced by the fact that vorticity represents rotation for rigid body, people believe it also works in fluid flow. However, not matching experiment results, this confusion drove scientists to look for better methods to describe vortex. According to Dr. Liu's classification, all methods applied to detect vortex can be categorized into three generations. The vorticity based method is classified as the first generation. Methods relying on eigenvalues of velocity gradient tensor are considered as the second generation. Although so many methods appeared, people still think vorticity is vortex since vorticity theory looks perfect in terms of math and all other methods are only scalars and unable to indicate swirl direction. Recently, Dr. Liu innovated a new vortex identification method called Liutex. Liutex, a vector quantity, which is regarded as the third-generation method, not only overcomes all previous methods' drawbacks, but also has a clear physical meaning. Direction of Liutex represents the swirl axis of rotation and its strength is equal to twice of angular speed. In this paper, we did a correlation analysis between vorticity and Liutex based on a DNS case of boundary layer transition. The result shows the correlation between vorticity and Liutex is almost zero, which demonstrates the idea that using vorticity to detect vortex is lack of scientific foundation. As a result, vorticity is not vortex.

**Keywords:** Liutex, vorticity, correlation, turbulence.

# Influence of Intrinsic Currents of Pyramidal Cells on the **Weak PING Rhythm**

#### Denis G. Zakharov

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The human brain demonstrates electrical oscillations of various frequency ranges. All of these oscillations are associated with a number of cognitive tasks. Here we will focus on the study of a weak (clustered) gamma rhythm (20-80 Hz). Typically, in the cortex, the gamma oscillations appear in the neuronal networks consisting of populations of excitatory pyramidal cells and inhibitory interneurons. This is so called PING (Pyramidal INterneuronal Gamma) oscillations. The weak gamma oscillations are a case when the pyramidal cells fire in some synchronous clusters producing "collective" rhythm by alternating firing clusters. We will consider the dependences of characteristics of the cluster states (mainly the number of clusters) on the intrinsic ionic currents of the PY cells (AHP-and m-currents). Actually, different number of clusters means different level of gamma oscillations coherence.

**Keywords:** Cluster syncronization, PING, gamma rhythms, Spike frequency adaptation, M-current, Multiple timer scales.

# Session title: Mathematical analysis of models arising from biology and physics

# Asymptotic behavior of solutions for a parabolic free boundary problem with nonlinear gradient absorption

#### **Zhengce Zhang**

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In this talk, we consider the free boundary problem for a parabolic equation with nonlinear gradient absorption. It is well known that global existence or blowup of solutions of nonlinear parabolic equations depends on which one dominating the model, the source or absorption, and on the absorption coefficient for the balance case of them. The aim of the paper is to study the influence of exponents of source and absorption, initial data and free boundary on the asymptotic behavior of solutions. At first, the ecological meaning of this model is explained by deriving the equation and the free boundary condition. Then, local existence and uniqueness are discussed, and the continuous dependence on initial data and comparison principle are proved. Furthermore, the finite time blowup and global solution are given by constructing sub- and super-solutions. In the different ranges of exponents and initial conditions, finite time blowup solutions, global fast solutions and global slow solutions were classified. Finally, the problem with double free boundaries was also discussed.

# **Vortex Identification for Study of Flow Past** Stationary and Oscillating Cylinder

#### Weiwen Zhao, Decheng Wan

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In this paper, vortex identifications for turbulent flows past an stationary and oscillating cylinder is performed on the three-dimensional velocity field obtained by detached-eddy simulation. The Reynolds number of the flow based on the cylinder diameter is 10,000. For the oscillating case, the moving boundary and the motion of the cylinder is archived by Arbitrary Lagrangian Eulerian method. Three different vortex identification methods, namely the Eulerian Q-criterion, Lambda2-criterion and the Rortex/Liutex are presented to understand the coherent turbulent flow structures. Quantitative flow variables such as drag and lift coefficients, pressure on the cylinder surface, velocity and vorticity profile are also presented.

**Keywords:** Flow structure interaction, vortex-induced vibration, detached-eddy-simulation.

### The identification of tip leakage vortex of an axial flow waterjet pump by using Omega method and Liutex

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The tip leakage vortex (TLV) is a common phenomenon in the field of rotating machinery. The formation and evolution of TLV can lead to some adverse effects, such as the interfering with the main flow, the occurrence of vortex cavitation, etc. This paper investigates the structure of TLV in an axial flow waterjet pump by using two vortex identification methods, namely the Omega method and the Liutex. These two criterions can capture the primary characteristics of the vortex structures with different vorticity strength in the flow passage and the tip gap region. As the cavitation number decrease, the tip leakage cavitation and the sheet

cavitation on the suction surface of the rotor blade present evident interactions with the TLV. On one hand, the TLV entrains the sheet cavitation and the tip leakage cavitation. On the other hand, the location and the shape of the TLV are dramatically influenced by the cavitation

Keywords: Omega method, Liutex, Tip leakage vortex, caviation, Axial flow waterjet pump.