A Nonlinear Time Series Expansion of the Logistic Chaos

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Abstract. The Weierstrass function derived from an exact chaos solution to the logistic map is firstly introduced, and a nonlinear time series expansion is proposed for the logistic chaos with a 2π -period in order to compose an infinite sum of trigonometric nonlinear functions. The nonlinear functions are computed by the algorithm given as a method without the accumulation of round-off errors in iterating the functions to minimize an error function for the expansion. Finally, it is shown that the logistic chaos is decomposed into a nonlinear time series by the proposed time series expansion, and the expansion generates the time series of 1/f noise depending on the power spectrum. **Keywords:** Logistic chaos, Chaos solution, Nonlinear time series, Time series expansion, Power spectrum, 1/f noise.

1 Introduction

In the field of nonlinear science, traditional and modern approaches to the time series analysis, which are based on statistics and the theory of dynamical systems, have been discussed, using a large number of data sets taken from various fields, such as biology, geophysics, economics and social sciences [3, 6]. As is well known, the Fourier series expansion decomposes periodic functions or periodic signals in terms of an infinite sum of simple oscillating functions, and has been applied to finding an approximation for original problems as harmonic analysis [5]. In addition, a large variety of analytic methods has been available for the quantification of nonlinear dynamics recorded in time series, which are some of the most prominent nonlinear properties. Recently, bifurcations, the Lyapunov exponents and fractal dimensions have been applied to the analysis of nonlinear dynamics, nonlinear prediction, noise reduction and climate records [15, 16].

On the other hand, the concept of chaos [12, 13] has been intensively studied, and has influenced thinking in many fields of science, since chaotic systems have shown rich and surprising structures with irregular behavior [14]. A most direct link between chaos and real time series is the nonlinear time series analysis of dynamical systems. The chaos theory has produced a wealth of powerful methods for the analysis of time series in nonlinear dynamical systems. In the meantime, the author has considered the fractal curves describing the Weierstrass function, which are obtained from exact chaos solutions to chaos maps [9, 10], and has proposed an algorithm without the accumulation of round-off errors by iterating chaos maps [8, 17]. The aim of this paper is to propose a nonlinear time series expansion for analyzing the logistic chaos [7], to show a

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relationship between the time series and 1/f noise found widely in nature [1, 4, 11], and to apply the proposed time series expansion to a generation of 1/f noise.

2 The Logistic Chaos and the Weierstrass Function

For simple functions of a nonlinear time series expansion proposed in Section 3, firstly we introduce an exact chaos solution;

$$x_n = \cos(C2^n), n = 0, 1, 2, \cdots,$$
 (1)

where a real number $C \neq \pm m\pi/2^{l}$ with finite positive integers $\{l, m\}$ to the logistic map $x_{n+1} = 2x_n^2 - 1$, and another solution $y_n = \sin(C2^n)$ to the chaos map $y_{n+1} = 2y_n(1 - 2y_{n-1}^2)$ [9]. From the solution (1), we derive the following Weierstrass-like function;

$$x(t) = \cos(2^n t) \tag{2}$$

with time t > 0, which gives fractal curve [10], and have a generalized function as

$$x(t) = \cos(p^n t), \tag{3}$$

and in a discrete form;

$$x(t_i) = \cos(p^n t_i), \tag{4}$$

where *p* and *t_i* are positive integers. Therefore, (3) is a Weierstrass function as $n \rightarrow \infty$, since x(t) is continuous but not differentiable anywhere. For example, time series of $x(t_i) = \cos(3^n t_i)$ are illustrated in Figure 1, and it is found that the time series have chaotic behaviors.

3 A Nonlinear Time Series Expansion

The well-known Fourier series expansion for a given periodic continuous function f(t) has been represented by

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(n\,\omega t) + b_n \sin(n\,\omega t) \right),\tag{5}$$

where ω is angular frequency, and we propose a nonlinear time series expansion for a periodic continuous function g(t) with a period 2π as follows;

$$g(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(p^n t) + b_n \sin(p^n t)),$$
(6)



Fig. 1. Time series of $x(t_i) = cos(3^n t_i)$ for n = 1, 5 and 10.

here p is a positive integer, and

$$\frac{a_0}{2} = \frac{1}{2\pi} \int_0^{2\pi} g(t) dt,$$
(7)

$$a_{n} = \frac{1}{\pi} \int_{0}^{2\pi} g(t) \cos(p^{n}t) dt,$$
(8)

$$b_n = \frac{1}{\pi} \int_0^{2\pi} g(t) \sin(p^n t) dt.$$
 (9)

Simple functions $\cos(p^n t)$ and $\sin(p^n t)$ in (6) are orthogonal, and then g(t) is proposed as a nonlinear expansion since the linear coefficient $n\omega$ in (5) corresponds to the nonlinear coefficient p^n in (6) with respect to n. At $t = t_i (i = 0, 1, 2, \dots, N)$ with the number N of time series in a 2π -period by dividing evenly into N intervals, (6) is exactly given by

$$g(t_i) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(p^n t_i) + b_n \sin(p^n t_i)),$$
(10)

where the coefficients $\{a_0, a_n, b_n\}$ in (6) and (10) are obtained by (7) - (9). Here, we introduce the following correction function for the logistic chaos time series $X_i \equiv \cos(C2^i), i = 0, 1, 2, \dots, N$ of (1) as

$$y_i = X_i - ai, a \equiv (X_N - X_0) / N,$$
 (11)

to have a periodicity, that is, a 2π -period at $y_0 = y_N = 0.0$ (see Figure 2), and define an error function;

$$\varepsilon \equiv \sqrt{\sum_{i=1}^{N} (y_i - g(t_i))^2 / N}$$
(12)

for minimizing the ε between the logistic chaos data y_i and the time series expansion $g(t_i)$ given by (10).

4 Numerical Examples

For the iterative calculation of the logistic chaos time series $X_i \equiv \cos(C2^i)$ without the accumulation of round-off errors, we introduce the algorithm [8] by setting;

$$C \equiv (l_i / m)\pi \tag{13}$$

and

$$l_{i+1} \equiv 2l_i \pmod{2m} \tag{14}$$

with an integer l_i and a large prime number *m*, and choose the following arbitrary three cases for the data X_i ;

Case 1

$$(l_0, m) = (167852967387, 31574166101), \tag{15}$$

Case 2

$$(l_0, m) = (8754681, 751234570907), \tag{16}$$

Case 3

$$(l_0, m) = (62547845, 784301365553) \tag{17}$$

with the arbitrary initial integer l_0 of l_i . Then, we can obtain the logistic chaos time series X_i without the accumulation of round-off errors in the iteration.

Next, for the calculation of simple functions $\cos(p^n t_i)$ and $\sin(p^n t_i)$ in the expansion (10), we use the algorithm by setting;

$$t_i \equiv (l_i / m)\pi \tag{18}$$

and

$$l_{i+1} \equiv pl_i \pmod{2m} \tag{19}$$

with $i = 0, 1, 2, \dots, N$ and a small prime number m = N - 1 to have the 2π period at i = N in (10). Thus, we find the optimal integer p and the optimal initial value l_0 of l_i to get a minimal $\varepsilon \approx 10^{-16}$, as an optimization problem, by iterating (12) and introducing PSO (Particle Swarm Optimization) for the highspeed optimization [2]. Then, the resultant nonlinear time series expansions with n=100 terms of $\cos(p^n t_i)$ and $\sin(p^n t_i)$ are given as

Case 1

$$g(t_i) = \frac{a_0}{2} + \{a_1 \cos(54t_i) + \dots + a_{100} \cos(54^{100}t_i) + b_1 \sin(54t_i) + \dots + b_{100} \sin(54^{100}t_i)\}, (20)$$

Case 2

$$g(t_i) = \frac{a_0}{2} + \{a_1 \cos(89t_i) + \dots + a_{100} \cos(89^{100}t_i) + b_1 \sin(89t_i) + \dots + b_{100} \sin(89^{100}t_i)\}, (21)$$

Case 3

$$g(t_i) = \frac{a_0}{2} + \{a_1 \cos(34t_i) + \dots + a_{100} \cos(34^{100}t_i) + b_1 \sin(34t_i) + \dots + b_{100} \sin(34^{100}t_i)\},$$
(22)

where the coefficient p^n in (20) – (22) corresponds to a higher frequency than that of the $n\omega$ in (5) [7]. The time series y_i and $g(t_i)$ are illustrated in Figure 2, and the optimal parameters $\{p, l_0\}$ and \mathcal{E} are shown for each case.



(a) Case 1: $(p, l_0) = (54, 70), \ \varepsilon = 6.08 \times 10^{-16}$







Fig. 2. The chaos data y_i (11) and the expansion $g(t_i)$ (10).

The power spectra of $g(t_i)$ (20)-(22) are represented for Cases 1-3 in Figure 3, and it is found that all the Cases have a flat average value, and show a property like white noise, that is, the logistic chaos time series has a property of white noise in terms of power spectra obtained by the numerical iteration without the accumulation of round-off errors.



Fig. 3. Power spectra of $g(t_i)$ (20)-(22).

Then, if we set the coefficients $\{a_n, b_n\}$ to have a property of 1/ f noise for Cases 1-3 in (20)-(22), we obtain the power spectra shown in Figure 4(a), and the time series of Cases 1-3 are illustrated in (b)-(d) of Figure 4, respectively. Here, it is interesting to note that the time series (b)-(d) of 1/ f noise in Figure 4 are generated by iterating the expansions (20)-(22), which are constructed on the basis of chaos, and have no accumulation of round-off errors in the iterative calculation.



(d) Case 3 in (a)

Fig. 4. Three 1/f noises obtained by setting the coefficients a_n and b_n of Cases 1-3 in (20)-(22).

Conclusions

In this paper, a nonlinear time series expansion has been proposed for the time series of the logistic chaos, where the chaotic time series are obtained from the exact chaos solution to the logistic map by introducing the algorithm [8] without the accumulation of round-off errors caused by iterating the calculation of the chaos solution. Here, the algorithm is used for simple functions $\cos(p^n t_i)$ and

 $\sin(p^n t_i)$ in the nonlinear time series expansion (10). As a result, it is shown

that the time series of the logistic chaos have a property of white noise in the power spectrum, and the expansions (20) - (22) generate 1/ f noise by setting the coefficients a_n and b_n . Therefore, the proposed nonlinear time series expansion

based on chaos would be applied to the analysis of nonlinear time series and the generation of 1/f noise.

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Simulation of MA(1) Longitudinal Negative Binomial Counts and Using a Generalized Methods of Moment Equation Approach

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Abstract

Longitudinal count data often arise in financial and medical studies. In such applications, the data exhibit more variability and thus the variance to mean ratio is greater than one. Under such circumstances, the negative binomial is more convenient to be used for modeling these longitudinal responses. Since these responses are collected over time for the same subject, it is more likely that they will be correlated. In literature, various correlation models have been proposed and among them the most popular are the autoregressive and the moving average structures. Besides, these responses are often subject to multiple covariates that may be time-independent or time-dependent. In the event of time-independence, it is relatively easy to simulate and model the longitudinal negative binomial counts following the MA(1) structures but as for the case of time-dependence, the simulation of the MA(1) longitudinal count responses is a challenging problem. In this paper, we will use the binomial thinning operation to generate sets of MA(1) non-stationary longitudinal negative binomial counts approach. Keywords:

Negative Binomial, Longitudinal, Moving Average ,Binomial thinning, Stationary, Nonstationary, Generalized method of moments

1 Introduction

In today's era, longitudinal data has become extremely useful in applications related to the health and financial sectors. It constitutes of a number of subjects that are measured over a specified number of time points. Since these measurements are collected for a particular subject on a repetitive basis, it is more likely that the data will be correlated. The correlation structures may be following autoregressive, moving average, equi-correlation, unstructured or any other general autocorrelation structures[4][5]. Moreover, in longitudinal studies, the responses are influenced by many factors such as in the analysis of

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CD4 counts, the influential factors are the treatment, age, gender and many others. In order to estimate the contribution and the significance of each of these factors towards the response variable, it is important to transform the data set-up into a regression framework. In literature, the regression parameters have been estimated by various approaches. Initially, the method of Generalized Estimating equations (GEE) were developed but it fails under misspecified correlation structure particularly under the independence correlation structure [5]. Thereafter, Prentice and Zhao [2] developed a Joint Estimation approach to estimate jointly the regression and correlation parameters and yielded more efficient regression estimates than the GEE approach but the joint estimation is based on higher order moments. Their approach is also based on the working correlation structure but the presence of these high order moments dilute the misspecification effect and boost the efficiency of the estimates. On the other hand, Qu and Lindsay [3] developed an adaptive quadratic inference based Generalized Method of Moments (GMM) approach where they assumed powers of the empirical covariance matrices as the bases. These bases are then used to form score vectors or moment estimating equations and thereafter, they were combined to form a quadratic function in a similar way as the GMM approach. This approach of analyzing longitudinal regression models has so far been tested on normal, Poisson data [3] but has not yet been explored in negative binomial correlated counts data. In this paper, our objectives are to develop the moment estimating equations based negative binomial model, construct the quadratic inference function and then obtain the regression estimates by maximizing the function. However, one challenging issue is that since the negative binomial model is a two parameter model (that is, depending on the mean and over-dispersion parameter), it implies that we will require higher order moments. This estimation approach will be tested via simulations on MA(1) stationary and non-stationary negative binomial counts. The organization of the paper is as follows: In the next section, we will review the negative binomial model along with its MA(1) Gaussian autocorrelation structure and the adaptive GMM approach following Qu and Lindsay [3]. In section 3, we will develop the estimating equations for the negative binomial model followed by simulation results.

2 Negative Binomial model

Longitudinal data comprise of data that are collected repeatedly over t = 1, 2, 3, ..., T time points for subjects i = 1, 2, 3, ..., I. Thus any i^{th}

random observation at t^{th} time point will have a representation of the form

 y_{it} . The negative binomial model for y_{it} is given by

$$f(y_{it}) = \frac{\Gamma(c^{-1} + y_{it})}{\Gamma(c^{-1})y_{it}!} \left(\frac{1}{1 + c\theta_{it}}\right)^c \left(\frac{c\theta_{it}}{1 + c\theta_{it}}\right)^{y_{it}}$$

with $E(y_{it}) = \theta_{it} = \exp(x_{it}^T \beta)$ and $Var(y_{it}) = \theta_{it} + c\theta_{it}^2$, c > 0 where in notation form,

$$y_{it} \sim NeBin(\frac{1}{c}, c\theta_{it})$$

given a $p \times 1$ vector of covariates x_{it}^T and vector of regression parameters β of the form $\beta = [\beta_1, \beta_2, ..., \beta_p]^T$, $y_i = [y_{i1}, y_{i2}, ..., y_{it}, ..., y_{iT}]^T$ and $\theta_i = [\theta_{i1}, \theta_{i2}, ..., \theta_{it}, ..., \theta_{iT}]^T$.

Since these counts y_{it} are collected repeatedly over time, it is more likely that y_{it} will

be correlated over time. In this paper, we will assume that the simulated y_{it} set of response variables come from the family of MA(1) Gaussian autocorrelation structure. The derivation of the MA(1) stationary negative binomial counts follows from McKenzie binomial thinning process[1]. However, the derivation of the MA(1) non-stationary correlation structure has not yet appeared in statistical literature. In the next section, we provide an in-depth derivation of the MA(1) non-stationary Gaussian autocorrelation structure.

3 MA(1) Non-Stationary Gaussian autocorrelation Structures

In the non-stationary set-up, the mean parameter at each time point will differ as the covariates are time-dependent, that

$$\theta_{i1} \neq \theta_{i2} \neq ... \theta_{it} \neq ... \theta_{iT}$$

Following McKenzie[1], we set up the framework to generate the MA(1) non-stationary Gaussian autocorrelation structure. The binomial thinning process assumes that

$$y_{it} = \alpha_{it} * d_{i,t-1} + d_{it}$$

where

$$d_{it} \sim NeBin(\frac{1}{c}, \dot{c}\theta_{i1}), \alpha_{it} \sim Beta(\frac{\rho}{c}, \frac{1-\rho}{c}) \text{ and,}$$
$$\alpha_{it} * y_{i,t-1} = \sum_{j=1}^{y_{i,t-1}} b_j(\alpha_{it}) = z_{it},$$
$$prob[b_j(\alpha_{it})=1] = \alpha_{it}, prob[b_j(\alpha_{it})=0] = 1 - \alpha_{it} \text{ and}$$

$$\dot{c} = \frac{c(1+\rho+2\rho^{2}+c+2c\rho+c\rho^{2})}{1+\rho^{2}+c+c\rho}$$

That is the conditional distribution of $\alpha_{it} * d_{i,t-1}$ follows the binomial distribution with parameters d_{it-1} and α_{it} . Under these assumptions, it can be proved that $y_{it} \sim NeBin(\frac{1}{c}, c\theta_{it})$ and the set of $y_i = [y_{i1}, y_{i2}, ..., y_{it}, ..., y_{iT}]^T$ follows the MA(1) structure. Under these distributional assumptions, we note that the covariance between y_{it} and y_{it-k} is given by $\frac{\rho \theta_{i,t-k}}{1+\rho} + \dot{c} \frac{\rho \theta_{i,t-k}^2}{(1+\rho)^2}$ for k = 1 and for other lags, the covariance does not exist.

4. Simulation of MA(1) Non –Stationary NB counts

The simulation process will follow from the binomial thinning operation explained in the previous section with $\theta_{it} = \exp(x_{it}^T \beta)$, that is we need to provide a given set of covariate designs and a set of regression vector β that respects the dimension of the covariate matrix. Note that for the stationary case, the covariate matrix will be time independent while for the non-stationary, the covariate design will be time-dependent. As such, we assume for the non-stationary case the following designs, Design A

$$x_{ii1} = \begin{cases} -0.5t \times rbinom(3,0.2), t = 1...\frac{I}{4} \\ t \times rpois(2), t = \frac{I}{4} + 1, ..., \frac{3I}{4} \\ 1.5 + t, t = \frac{3I}{4} + 1, ..., I \end{cases}$$

Design B

$$x_{it1} = \begin{cases} -0.5t \times \sin t, t = 1...\frac{I}{4} \\ \exp(t), t = \frac{I}{4} + 1, ..., \frac{3I}{4} \\ \cos t, t = \frac{3I}{4} + 1, ..., I \end{cases}$$

Design C

$$x_{it1} = \begin{cases} t, t = 1 \dots \frac{I}{4} \\ \ln(t), t = \frac{I}{4} + 1, \dots, \frac{3I}{4} \\ t - 1, t = \frac{3I}{4} + 1, \dots, I \end{cases}$$

and X_{it2} is generated from the Poisson distribution with mean parameter 2. In this way,

the mean parameter for each subject *i* will vary. Thus, for these set of covariates and initial estimate of the regression vector, dispersion parameter and correlation parameter, we generate MA(1) Negative Binomial random variables by first simulating the error components d_{it} , y_{it-1} and the thinning operation random variables $\alpha_{it} * y_{i,t-1}$. For

our simulation process, we will assume the values of $\beta = [1,1]^T$.

5. Estimation Methodology

Qu and Lindsay [3] have developed an estimation approach based Generalized Methods of Moments that do not require any assumption in the underlying correlation structure and do not require any estimation of the correlation parameter. In fact, Qu and Linsday [3] assumed a score vector that only needs the empirical covariance estimation matrix

$$V = \frac{1}{I} \sum_{i=1}^{I} (y_i - \theta_i) (y_i - \theta_i)^T ,$$
$$g = \begin{pmatrix} \sum_{i=1}^{I} D_i^T (y_i - \theta_i) \\ \sum_{i=1}^{I} \alpha^T D_i^T V (y_i - \theta_i) \end{pmatrix}$$

where D_i is the gradient matrix: $D_{it} = \frac{\partial \theta_{it}}{\partial \beta^T}$ and α is an orthogonal vector. The

calculation of the parameter α requires the conjugate gradient method [see Qu and Lindsay [3]]. In the context of the negative binomial model, the score vector g is defined as:

$$g = \begin{pmatrix} \sum_{i=1}^{I} D_i^T (f_i - \theta^*_i) \\ \sum_{i=1}^{I} \alpha^T D_i^T V (f_i - \theta^*_i) \end{pmatrix}$$

where the vectors $f_i = [y_i, y_i^2]^T$, $\theta^*_i = E[f_i] = [\theta_i, \theta_i + (c+1)\theta_i^2]^T$,

$$V = \frac{1}{I} \sum_{i=1}^{I} (f_i - \theta^*_i)(f_i - \theta^*_i)^T \text{ and}$$
$$D_i = [\frac{\partial \theta^*_i}{\partial \beta^T}, \frac{\partial \theta^*_i}{\partial c}] = [D_{i1}, D_{i2}, ..., D_{it}, ..., D_{iT}]^T \text{ where}$$

$$D_{it} = \begin{pmatrix} \frac{\partial \theta_{it}}{\partial \beta^{T}} & 0\\ \frac{\partial [\theta_{i} + (c+1)\theta_{i}^{2}]}{\partial \beta^{T}} & \frac{\partial [\theta_{i} + (c+1)\theta_{i}^{2}]}{\partial c} \end{pmatrix}$$

Using the score vector g, Qu and Lindsay [3] defined the objective function

$$Q(\beta,c) = g^T C^{-1} g$$

where C is the sample variance of g

$$\begin{pmatrix} \sum_{i=1}^{I} D_i^T V D_i & [\sum_{i=1}^{I} D_i^T V^2 D_i] \alpha \\ [\sum_{i=1}^{I} D_i^T V^2 D_i] \alpha & \alpha^T [\sum_{i=1}^{I} D_i^T V^3 D_i] \alpha \end{pmatrix}$$

By maximizing the objective function with respect to the unknown set of parameters, we obtain the estimating equation

$$\dot{Q}(\beta,c) = 2 \dot{g}^T C^{-1} g$$

with $\dot{g} = \left[\frac{\partial g}{\partial \beta^T}, \frac{\partial g}{\partial c}\right]^T$. Since the above estimating equation is non-linear, we solve

the equation using the Newton-Raphson procedure that yields an iterative equation of the form

$$\begin{pmatrix} \hat{\beta}_{r+1} \\ \hat{c}_{r+1} \end{pmatrix} = \begin{pmatrix} \hat{\beta}_r \\ \hat{c}_r \end{pmatrix} - [\dot{Q}(\beta,c)]_r^{-1} [\dot{Q}(\beta,c)]_r$$

where $[\dot{Q}(\beta,c)] = 2 \dot{g}^T C^{-1} \dot{g}$ is the double derivative hessian part of the score function and this is being used for calculating the variance of the regression and overdispersion parameters. As illustrated by Qu and Lindsay [3], this method yields consistent and efficient estimators and tends towards asymptotic normality for large sample size.

6. Results and Conclusion

Following the previous sections, we have run 10,000 simulations for each of the sample sizes I = 20,50,100,200,500 based on the different covariate designs for the non-stationary set-ups. Note that for the stationary case, the mean is held constant at all time points whilst for non-stationary, the mean varies with the time points given the time-dependent covariates. The table provides the simulated mean estimates of the regression parameters along with the standard errors in brackets.

Ι	Design A	Design B	Design C
20	0.9919;1.0010	1.0121;0.9987	0.9956;1.0013
	(0.1351;0.2120)	(0.1401;0.1971)	(0.2212;0.1898)
50	1.0110;0.9978	0.9919;0.9995	0.9982;1.0121
	(0.1022;0.1762)	(0.1211;0.1881)	(0.1580;0.1)

100	0.9982;0.9995	1.0101;0.9961 (0.0754:0.1052)	0.9988;1.0015 (0.0889:0.1010)
200	1.0012;1.0005	0.9992;0.9992	1.0042;1.0141
	(0.0661;0.0991)	(0.0762:0.0975)	(0.0562;0.0888)
500	0.9999;1.0001	0.9992;0.9993	0.9978;1.0010
	(0.0552;0.0808)	(0.0432;0.0652)	(0.0466;0.0762)

Based on the simulation results, we note that the estimates of the regression parameters are close to the population values and as the sample size increases, the standard errors of the regression parameters decrease which indicates that the estimates are consistent and efficient. However, we have remarked a significant number of failures in the simulations as we increase the sample size. These failures were mainly due to ill-conditioned nature of the double derivative Hessian matrix. To overcome this problem in some simulations, we have used the Moore Penrose generalized inverse method in R (ginv in Library MASS) to perform the iterative procedures. Overall, the generalized method of moments estimation technique is a statistically sound technique but in terms of computation, it may not always be reliable.

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Learning dynamical regimes of Solar Active Region via homology estimation

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Abstract. The development of numerical methods of mathematical morphology and topology gives us opportunity to analyze various structures on the plane and in space. In particular they can be used to analyze the complexity of the image by estimating the variation of the number of connected structures and holes depending on the brightness level. Alternate sum of this numbers gives topological invariant Euler characteristic. The other approach to estimation this characteristic is persistent homology calculation at the different sub level sets. It turned out that the application of these ideas to the active regions of the Sun magnetograms allowed diagnostic changes in different dynamic regimes connected with sun flares.

Keywords: Topological persistence, mathematical morphology, dynamical regimes detections, Sun Active Region, homology.

1 Introduction

Large solar flares are the most dramatic results of the evolution of the magnetic fields in sunspots. The energy of such flare reaches 10^{32} erg and the peak power reaches about 10^{29} erg/sec. For the most powerful X flares energy density reaches 10^{-4} W/m².

Energetic flares which are occurred near the center of the solar disk could make a disastrous damage of the terrestrial and space equipment. First of all, there are failures and crashes of space crafts on geocentric orbits, see in Karimova et al. [1], increase in background radiation at altitudes of manned space crafts, radio blackout caused by magnetic storms, induced currents in pipelines that reach hundreds of amperes, failures in automatic control systems in metropolitan areas and many others, see in Pulkkinen [2].

Large flares tend to occur in big groups of spots, so called active regions (ARs) of the Sun. Flaring ARs may contain more than a dozen spots of different polarities forming a topologically complex spatial configuration of the magnetic fields, seen in Longcope [3] and in Borrero and Ichimoto [4]. The problem of early prediction of the X-ray flares is a practically important scientific task. It is complicated by the fact that there is no correct theoretical assumptions about the solar flares appearance so far. Existing forecast methods are based largely on the rich observation phenomenology. The most common approach associate the flares with variations of space complexity of the AR, i.e. a complex of geometric and/or kinematic features observed in dynamic scenarios of the AR. the most of them were suggested in 1972 by Smith [5]. Many contemporary publications on flare precursors investigation (see, for example Cui et al. [6], Falconer et al. [7] and Mason and Hoeksema [8]) also rely on

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the characteristics describing some changes in the magnetic structures. These changes are traced either in magnetogram's patterns or in the features of scalar or vector fields reconstructed from digital images. It is believed that the precursors are produced by the dynamics of new magnetic fields emerging inside or in the neighborhood of the AR, seein Lites[9]. Sometimes such flows can be observed directly, as described in Magara [10], but, in general, their detection in monitoring mode is a separate and challenging problem, see in Knyazeva *et al.* [11].

In this paper for describing topological complexity of magnetic field we suggest to use methods comes from mathematical morphology and algebraic topology. The main idea of this approach is to consider magnetogramm as a 3D random field. We consider the changes in topology of magnetogramm as a changes in behaviour of peaks and dips of random field.

2 Mathematical morphology

Estimation of morphological functionals for physical fields are based on the stochastic-geometry methods developed by Adler [12] and Worsley [13]. These were begun with the pioneering work of Rice [14], who proposed to study random processes by considering the distributions of plots beyond some specified level. The mean time a plot spends above the specified level, i.e., the duration of the excursions, and the number of excursions per unit time serve as useful statistics in this case. For two dimensional fields is considered so-called excursion sets. This is a set formed by the values which exceeds the specified values . On the excursion set Minkowski functionals could be estimated, see in Adler [12], and Worsley [13]. Euler characteristic (EC or χ) the main of them. The formal basis based on Morse theory see in Bobrowski[15] and Matsumoto [16].

The magnetograms represent a matrix containing values of the line-of-sight magnetic field. The main idea is separating the magnetograms into a set of binary images with the selected steps. Let's consider an excursion set

$$A_u = \{ \mathbf{x} \in W : B_z(\mathbf{x}) \ge u \}$$

$$\tag{1}$$

of the field in a compact region W, formed by the pixels x?W where the magnetic field $B_z(x)$ exceeds a specified level u. We mark these pixels black. This makes it possible to translate each magnetogram into a set of black and white images, one for each selected level. At each level be can define the number of connected components (islands) m0 and holes in the islands m1. Then, it can be shown by Adler (1981) that:

$$\chi(A_u) = m_0 - m_1. \tag{2}$$

It could be shown that $\chi(A_u)$ measures the topological complexity of the field on the excursion set u. It is not difficult to estimate the Euler characteristic for each of these levels. This quantity is a measure of the complexity of the magnetic-field topology. So for the sequence of magnetogram we will have a sequence of EC for each excursion set. This allows us to trace the changing in topology of magnetic field as a changes in EC . The main drawback in this approach that we have EC for each excursion set, so we need to analyse many evolution of EC at each level or choose previously level.

3 Persistence homology

The second approach to estimate the Euler characteristic is connected to persistent homology [17] and a technique based on deep relations of persistence diagrams with the Hausdorff measures of singular points of random fields [15]. In this case, the main contribution to the estimates gives a topography of neighbourhoods of the big field excursions and correlations of extrema of the field on a large scale. The structure of the field is determined by the content of the local neighborhoods for the maxima and minima: how many and at which level peaks or dips appear which are close to the given maximum or minimum. Also we would like to know up to which level field maxima (minima) are isolated in a some local neighborhood. We can measure a life time of each isolated peak as the length of the interval or barcode on which it is separated from others. It is usefull to draw it on the plane using the beginning and the end of the barcode as point coordinates. As the result we obtain a set of points which lie above the diagonal that corresponds to barcodes of the zero length. This graph is called a persistence diagram. It is convenient to give some simple structure at the neighborhood of the maximum — so-called simplicial structure.

The computation of Betti numbers comes from algebraic topology and developed for simplical complexes. There are basis of the relevant definitions in book of Edelsbrunner and Harer [17].. The incremential algorithm for computing homology which we used in our work could be found in the article of Delfinado and Edelsbrunner [18]. It consists with two sequential steps: filter construction of simplices (for two-dimensional images the simplex is a vertex, an edge or a triangle) and computing the Betti numbers on the created filtration. Let f(x, y) is a value at pixel (x, y).For the filter construction we need to determine the function value for each of simplices. In order to do this we associate each pixel (x, y) of the image with the vertex. We define the value for the remaining simplices by assigning the maximum of values between their vertices. Now we describe the algorithm for the filter construction. First we sort all vertices (pixels of the image) in increasing order of their function value F(v) (i. e. the intensity level of the corresponding pixels) and create a sequence

$v_1, v_2, v_3, \ldots, v_n$.

Let us further assume that if two vertices have the same value of the function F then the vertex, which is higher or to the left of the second vertex on the image, is located closer to the beginning of the sequence (3). Next, we iterate through all elements of the ordered sequence and add each of them to the filter. At the same time, attaching the new vertex to the filter we add all edges and all triangles that can be generated by vertices which we already have in the filter and the new vertex. A condition for creating the edge or the triangle is presence of two neighboring vertices for the edge and three neighboring vertices

for the triangle (such a way that no two edges cross each other). As a result we obtain the filter, the sequence of simplices,

$$s_1, s_2, s_3, \ldots, s_n$$
.

such that the simplices there are sorted in increasing order of their value $F(s_j)$. To distinguish topological spaces based on the connectivity of n-dimensional simplicial complexes are used Betti numbers. Informally, the k- th Betti number refers to the number of k-dimensional holes on a topological surface. B0 is the number of connected components, B_1 is the number of one-dimensional or "circular" holes. In our case there are only B_0 and B_1 . We can compute the B_0 and B_1 numbers by processing the simplices in the filter and keeping track of changes in connectivity of the obtaining set. Here, the basic data structure is the Union-Find data structure. This structure supports two operations, namely Find(i) and Union(i, j). Find(i) returns the number of connected components that contain i. If i and j belongs to different components, then Union(i, j) operation merge them in one.

Now we can compute the Betti numbers by processing the simplices in the filter and keeping track of changes in connectivity of the obtaining set. For computing B0 we processed simplices in the direct order. If we add vertex we add components, if there is an edge in filtration we need to check if the vertexes of edge belongs to different components, if belong than the number of components decrease by one and we merge components in other case nothing happens. To compute Holes or B1 we use the same algorithm applying it to a dual graph. In the dual graph to each vertex corresponds the triangle of the initial graph, to each triangle corresponds the vertex in the initial graph and to each edge corresponds the dual edge. We add at the end of the filter with the value minus infinity. After that we apply the algorithm described above with one small correction: we go backwards through elements of the filtration and compute the persistence for the dual graph. As a simple example at Fig 1a we represent several steps of filtration processing for 6x6 matrix, at Fig 1b marked all the holes in test matrix.



Fig. 1. First steps of incremental algorithm B_0 or components (a) and B_1 or holes (b) computing

This algorithm can be supplemented by computing the so-called persistence of connected components. By the persistence we mean the life time of the corresponding connected component, i. e. a range of intensity values in which the given component exists. If vertices of the current edge belong to different connected components, then after merging them into a single component we suppose that the component, which appeared later than another, disappears ("dies"). In that way we can keep track of "birth" and "death" of connected components at the intensity levels. The same true for holes. If we sum all life length for B_0 and for B_1 and take difference of them we receive average value of the Euler characteristic,see Bobrowski [15]

$$\chi(\mathcal{B}) = L(B_0) - L(B_1) \tag{3}$$

4 Results

We used a time sequence of magnetograms of the full solar disk, obtained with the help of a Helioseismic and Magnetic Imager (HMI) tool, installed aboard the Space Observatory SDO, see Scherrer et al. [19]. The angular resolution of HMI data is $\approx 0.5''$ /pixel (it corresponds to a linear scale of about 500 km/pixel). The data represent a matrix of $4000 \times 4000 \text{ pixels}$ which contains the values of the flux density of the component $B_z(\mathbf{x})$ of magnetic field of the Sun, directed along the line-of-sight. A time interval between magnetograms was 720 seconds, and the noise level does not exceed 6 gauss. A fragment of 600×600 pixels containing the AR was cut from each magnetogram. For the specified 720 seconds time gap about 700 consecutive images of the same active region passing across the solar disk were available. We considered only the 60-degrees circular area about the center of the disk to avoid the significant geometric distortions. We used FI index of flare productivity to compare the variations to flare activity. Roughly speaking, it measures a weighted amount of energy produced by solar flares of various classes in the finite time interval. The flare classes FI were converted to numeric values in a standard way, namely the magnitudes of C class flares were not altered, for M class flares the magnitudes were multiplied by 10, for class X were multiplied by 100, and for B class were divided by 10 We present here the results of numerical experiments for two flare-active regions AR 11520 and AR 11158.

AR 11158 appeared near the center of the solar disk as a compact β -class bipolar group on February 12, 2011. Within a day it reached δ magnetic class and on 12 February produced a flare of class M6.6. A day later M2.2 flare followed, and, finally, on 15 February X2.2 flare occurred. After that activity of this AR actually stopped ,see in Sun *et al.*[20]. The dynamics of the Euler characteristic for the high levels of magnetic field strength is shown in Fig. 2 a). At Fig. 2 a) represents a behaviour of the persistence homology difference $B_0 - B_1$. The complexity of the field in Fig. 2 a) is growing for the fields of north and south polarities, anticipating an increase in flare productivity. Little depression could be seen before the big flare. For comparison, Fig. 2 b) shows the behaviour of the Euler characteristic obtained by the persistent homology. Here we note a depression in the EC graph preceding the phase of flare activity. The depression is the most obvious about a day before the X flare.



Fig. 2. The dynamics of EC for AR 11158: high levels of magnetic field strength (a). The dynamics of persistent homology b0-b1 (b)

AR 11520. This active region appeared on the Sun on July 8, 2012. It was immediately assigned to the class of complex large groups of δ -configuration with a possible high flare productivity. Initially, the region was a single large penumbra which contained many small spots of the opposite polarity. In the course of evolution it began quickly disintegrate into several compact regions. Against all expectations, the AR 11520 produced only four flares of M class and one flare X1.4 on 12 July. The last flare approximately corresponded to the localization of the group near the center of the solar disk. After that the AR 11520 flare activity stopped. At Fig. 3 a) dynamics of EC at high levels of magnetic field is shown , before the X flare strong depression could be seen. An Fig. 3 b the evaluations of the Euler characteristic for the AR 11520 obtained by the persistent homology are shown. Again we can see well marked variations in topological complexity of the field before the X flare.



Fig. 3. The dynamics of EC for AR 11520: high levels of magnetic field strength (a). The dynamics of persistent homology b0-b1 (b)

5 Conclusion

The main aim of the present work was to develop some topological approaches for the analysis of the magnetic field of the Sun which are oriented to the detection of pre-flare scenarios. The data are SDO/HMI magnetograms. Two of them were selected for the analysis AR 11520 and AR 11158. For these active regions the strongest flares of the class X far from the limb of the disk were observed. For the corresponding sequence of magnetograms we obtained time variations of the Euler characteristic. The EC was estimated in two ways. With the first approach, it is obtained as one of the Minkowski functionals computed on the excursion sets of the observed component of the magnetic field strength. The second way is based on the methods of computational topology. The persistence diagrams were used for the estimation of the sum of barcodes lengths for the first two Betti numbers. The alternating sum of this lengths might be considered as the averaged estimate of the Euler characteristic. In morphological approach for each magnetogram we computed the whole set of EC for each of excursion set, after that we need to specify some level of magnetic field and track evolution of EC of them. On the contrary, the persistent homology consider the full structure of the magnetic field of the AR.

The active regions under study demonstrate different dynamics which are tracked by patterns of the magnetic field. Typically significant variations of the Euler characteristic often precede the flares. It should be noted that the results presented in this paper confirm our earlier works obtained from the MDI/SOHO magnetograms. This fact slightly compensates for a lack of the adequate statistical sample restricted by the low level of the solar activity at the present time. Nevertheless, topological approaches satisfy the empirical considerations of the primary role of topological changes in the magnetic fields of active regions.

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Chaos at Cross-waves in Fluid Free Surface

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Abstract: The phenomenon of chaotic cross-waves generation in fluid free surface in two finite size containers is studied. The waves may be excited by harmonic axisymmetric deformations of the inner shell in the volume between two cylinders and in a rectangular tank when one wall is a flap wavemaker. Experimental observations have revealed that waves are excited in two different resonance regimes. The first type of waves corresponds to forced resonance, in which axisymmetric patterns are realized with eigenfrequencies equal to the frequency of excitation. The second kind of waves is parametric resonance waves and in this case the waves are "transverse", with their crests and troughs aligned perpendicular to the vibrating wall. These so-called cross-waves have frequencies equal to half of that of the wavemaker. The existence of chaotic attractors was established for the dynamical system presenting cross-waves and forced waves interaction at fluid free-surface in a volume between two cylinders of finite length. In the case of one cross-wave in a rectangular tank no chaotic regimes were found. Keywords: Cross-waves, Wavemaker, Fluid free sureface, Averaged systems, Parametric resonance, Chaotic simulation.

1 Introduction

The phenomenon of cross-waves generation in free-surface waves of a fluid confined in a rectangular tank with the finite depth and one wall as a flap wavemaker is rather known, Faraday, 1831, [3]. The waves may be excited by harmonic oscillations of wavemaker and depending on the vibration frequency both axisymmetric and non-symmetric wave patterns may arise. Experimental observations have revealed that waves are excited in two different resonance regimes. The first type of waves corresponds to forced resonance, in which axisymmetric patterns are realized with eigenfrequencies equal to the frequency of excitation. The second kind of waves is parametric resonance waves and in this case the waves are "transverse", with their crests and troughs aligned perpendicular to the vibrating wall. These so-called cross-waves have frequencies equal to half of that of the wavemaker, Faraday, 1831, [3]. To obtain a lucid picture of energy transmission from the wavemaker motion to the fluid free-surface motion the method of superposition, Lamé, 1852, [8], has been used. This method allows to construct a simple mathematical model, which shows how the cross-waves can be generated directly by the wavemaker. All previous theories have considered cross-waves problem applying the Havelock's, 1929, [2], solution of the wavemaker problem for a semi-infinite tank with an infinite depth and a radiation condition instead of zero velocity condition at the finite bottom.

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As the second task the phenomenon of deterioration of fluid free-surface waves between two cylindrical shells when the inner wall vibrates radially is considered in the present paper.

2 Approximation of Cross-waves in Rectangular Container

Let us theoretically consider the nonlinear problems of fluid free-surface waves which are excited by a flap wavemaker at one wall of rectangular tank of a finite length and depth. From the experimental observations, Krasnopolskaya, 2013, [6], we may conclude that the pattern formation has a resonance character, every pattern having its "own" frequency. Assuming that the fluid is inviscid and incompressible, and that the induced motion is irrotational, the velocity field can be written as $\mathbf{v} = \nabla \varphi$. Let us consider that patterns can be described in terms of normal modes with characteristic eigenfrequencies, we approximate free surface displacement waves, when the excitation frequency ω is twice as large as one of the eigenfrequencies, i.e. $\omega \approx 2\omega_{nm}$, and also is close to other eigenfrequency $\omega \approx \omega_{l0}$, as a function written in the form

$$\xi \approx \xi_{nm}(t) \cos \frac{n\pi x}{L} \cos \frac{m\pi y}{b} + \xi_{lo}(t) \cos \frac{n\pi x}{L} + \xi_{00}.$$
(2.1)
When $\xi_{nm}(t) = O(\varepsilon_1^{3/2})$, $\xi_{lo} = O(\varepsilon_1)$, $\xi_{lo} = O(\varepsilon_1)$ and $\varepsilon_1 = \frac{a\omega_{nm}^2}{g}$

Where *a* is an amplitude of wavemaker oscillations, *L* is the length, *b* is the width and *h* is the depth of the fluid container. Then a potential of fluid velocity $\varphi = \varphi_1 + \varphi_2 + \varphi_0$ as the solution of the harmonic equation and according to [5] has following components

$$\varphi_1 = \varphi_{nm}(t) \cos \frac{n\pi x}{L} \cos \frac{m\pi y}{b} \frac{\operatorname{ch}[k_{nm}(z+h)]}{\operatorname{ch}(k_{nm}h)} + \varphi_{lo} \cos \frac{n\pi x}{L} \frac{\operatorname{ch}[k_{lo}(z+h)]}{\operatorname{ch}(k_{lo}h)}.$$

$$\varphi_{2} = -\varepsilon_{1} \cos \omega t \sum_{j=1}^{\infty} \frac{4g h[1 - (-1)^{j}]}{\omega_{nm} \alpha_{0j} (\operatorname{th} \alpha_{0j} L) (j\pi)^{2}} \cos \frac{j\pi z}{h} \frac{\operatorname{ch} \alpha_{0j} (x - L)}{\operatorname{ch} \alpha_{0j}} - \varepsilon_{1} \dot{\xi}_{nm} \sin \omega t \frac{g \left[\frac{3}{2} \left(\frac{n\pi}{L} \right)^{2} + \frac{k_{nm} \operatorname{th} k_{nm} h}{h} \right]}{\omega_{nm}^{2} h \alpha_{0j} \alpha_{m0} \operatorname{th} \alpha_{m0} L (k_{nm} \operatorname{th} k_{nm} h)} \cos \frac{m\pi y}{b} \frac{\operatorname{ch} \alpha_{m0} (x - L)}{\operatorname{ch} \alpha_{m0}} - \varepsilon_{1} \dot{\xi}_{nm} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{0j} \alpha_{m0} \operatorname{th} \alpha_{m0} L (k_{nm} \operatorname{th} k_{nm} h)}{\omega_{m0}^{2} h \alpha_{0j} \alpha_{m0} \operatorname{th} \alpha_{m0} L (k_{nm} \operatorname{th} k_{nm} h)} \cos \frac{m\pi y}{b} \frac{\operatorname{ch} \alpha_{m0} (x - L)}{\operatorname{ch} \alpha_{m0}} - \varepsilon_{1} \dot{\xi}_{nm} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{0j} \alpha_{m0} \operatorname{th} \alpha_{m0} L (k_{nm} \operatorname{th} k_{nm} h)}{\omega_{m0}^{2} h \alpha_{0j} \alpha_{m0} \operatorname{th} \alpha_{m0} L (k_{nm} \operatorname{th} k_{nm} h)} \cos \frac{\omega_{m0}^{2} h \alpha_{m0} (x - L)}{\operatorname{ch} \alpha_{m0}} - \varepsilon_{1} \dot{\xi}_{mm} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{0j} \alpha_{m0} \operatorname{th} \alpha_{m0} L (k_{m0} \operatorname{th} k_{mm} h)}{\omega_{m0}^{2} h \alpha_{m0} (x - L)} - \varepsilon_{1} \dot{\xi}_{mm} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{m0} (x - L)}{\omega_{m0}^{2} h \alpha_{m0} (x - L)} - \varepsilon_{1} \dot{\xi}_{mm} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{m0} (x - L)}{\omega_{m0}^{2} h \alpha_{m0} (x - L)} - \varepsilon_{1} \dot{\xi}_{mm} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{m0} (x - L)}{\omega_{m0}^{2} h \alpha_{m0} (x - L)} - \varepsilon_{1} \dot{\xi}_{m0} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{m0} (x - L)}{\omega_{m0}^{2} h \alpha_{m0} (x - L)} - \varepsilon_{1} \dot{\xi}_{m0} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{m0} (x - L)}{\omega_{m0}^{2} h \alpha_{m0} (x - L)} - \varepsilon_{1} \dot{\xi}_{m0} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{m0} (x - L)}{\omega_{m0}^{2} h \alpha_{m0} (x - L)} - \varepsilon_{1} \dot{\xi}_{m0} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{m0} (x - L)}{\omega_{m0}^{2} h \alpha_{m0} (x - L)} - \varepsilon_{1} \dot{\xi}_{m0} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{m0} (x - L)}{\omega_{m0} (x - L)} - \varepsilon_{1} \dot{\xi}_{m0} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{m0} (x - L)}{\omega_{m0} (x - L)} - \varepsilon_{1} \dot{\xi}_{m0} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{m0} (x - L)}{\omega_{m0} (x - L)} - \varepsilon_{1} \dot{\xi}_{m0} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{m0} (x - L)}{\omega_{m0} (x - L)} - \varepsilon_{1} \dot{\xi}_{m0} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{m0} (x - L)}{\omega_{m0} (x - L)} - \varepsilon_{1} \dot{\xi}_{m0} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{m0} (x - L)}{\omega_{m0} (x - L)} - \varepsilon_{1} \dot{\xi}_{m0} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{m0} (x - L)}{\omega_{m0} (x - L)} - \varepsilon_{1} \dot{\xi}_{m0} \sin \omega t \frac{\omega_{m0}^{2} h \alpha_{m0} (x - L)}{\omega_{m0}$$

$$-\varepsilon_{1}\dot{\xi}_{nm}(t)\sin\omega t\sum_{j=1}^{\infty}\frac{g\left(\frac{n\pi}{L}\right)^{2}\left[1-(-1)^{j}\right]}{\omega_{nm}^{2}\alpha_{nj}(j\pi)^{2}\operatorname{th}\alpha_{nj}L(k_{nm}\operatorname{th}k_{nm}h)}\cos\frac{m\pi y}{b}\cos\frac{j\pi z}{h}\frac{\operatorname{ch}\alpha_{nj}(x-L)}{\operatorname{ch}\alpha_{nj}}$$

Where $\varphi_{nm}(t) = O(\varepsilon_1^{1/2})$ and $\varphi_{lo} = O(\varepsilon_1)$ Using kinematical free-surface boundary conditions, Krasnopolskaya, 2012, [5], $(\varphi_0)_z + (\varphi_1)_z + \xi(\varphi_0)_{zz} + \xi(\varphi_1)_{zz} + \xi^2(\varphi_1)_{zzz} + \xi(\varphi_2)_{zz} =$ $=\xi_{t} + (\varphi_{1})_{x}\xi_{x} + (\varphi_{1})_{y}\xi_{y} + (\varphi_{0})_{x}\xi_{x} + (\varphi_{2})_{y}\xi_{y} +$ $+(\varphi_1)_{xz}\xi\xi_x+(\varphi_1)_{yz}\xi\xi_y$, we may find that the amplitude of the resonant cross-wave mode is 2)

$$\xi_{nm}(t) = \frac{\xi_{nm}}{k_{nm} \operatorname{th} k_{nm} h} - \varepsilon_1 \xi_{nm} D \cos \omega t; \qquad (2.1)$$

when

$$D = \frac{1}{k_{nm} \operatorname{th} k_{nm} h} \left[\sum_{j} \frac{8g[1 - (-1)^{j}]}{\omega_{nm} h^{2} \alpha_{0j} (\operatorname{th} \alpha_{0j} L)} \right]_{0}^{L} \cos^{2} \frac{n\pi x}{L} \frac{\operatorname{ch} \alpha_{0j} (x - L)}{\operatorname{ch} \alpha_{0j} L} dx + \frac{g}{\omega_{nm} L} - \frac{g}{\omega_{nm} L} \int_{0}^{L} \left(\frac{n\pi}{L} \right) (x - L) \sin 2 \frac{n\pi x}{L} dx.$$

Applying the dynamical boundary condition

$$\begin{aligned} (\varphi_{0})_{t} + (\varphi_{1})_{t} + \xi(\varphi_{1})_{tz} + \xi^{2}(\varphi_{1})_{tzz} + (\varphi_{2})_{t} + g\xi + \\ + \left[(\varphi_{1})_{x}^{2} + (\varphi_{1})_{y}^{2} + (\varphi_{1})_{z}^{2} \right] + (\varphi_{1})_{x}(\varphi_{2})_{x} + (\varphi_{1})_{y}(\varphi_{2})_{y} + (\varphi_{1})_{z}(\varphi_{2})_{z} + \\ + (\varphi_{1})_{x}(\varphi_{0})_{x} + (\varphi_{1})_{z}(\varphi_{0})_{z} + (\varphi_{1})_{z}\xi(\varphi_{1})_{zz} + \\ + (\varphi_{1})_{x}\xi(\varphi_{1})_{xz} + (\varphi_{1})_{y}\xi(\varphi_{1})_{yz} = F_{0}(t), \end{aligned}$$

we can get for the resonant amplitude an equation of parametric oscillations Ω

$$\ddot{\xi}_{nm} + \omega_{nm}^2 \xi_{nm} - \frac{9}{16} \omega_{nm}^2 k_{nm}^2 \xi_{nm}^3 + \frac{3}{4} k_{nm}^2 \xi_{nm} \dot{\xi}_{nm}^2 + \varepsilon_1 D_2 \xi_{nm} \sin \omega t - \varepsilon_1 D_5 \dot{\xi}_{nm} \cos \omega t = 0.$$
(2.3)

We can write it for the rectangular tank with L = 50 m, h = 2.5 m, b = 6.8 m and for the wave numbers n = 40, m = 10 in the form

$$\ddot{\xi}_{nm} + \omega_{nm}^2 \xi_{nm} - \frac{9}{16} \omega_{nm}^2 k_{nm}^2 \xi_{nm}^3 + \frac{3}{4} k_{nm}^2 \xi_{nm} \dot{\xi}_{nm}^2 + 0.0478 \omega_{nm}^2 \xi_{nm} \sin \omega t - 0.0299 \omega_{nm} \dot{\xi}_{nm} \cos \omega t = 0.$$
(2.4)
Where $k_{nm}^2 = \left(\frac{n\pi}{L}\right)^2 + \left(\frac{m\pi}{b}\right)^2$, the frequency is $\omega_{nm} = 2\pi 1.143$ Hz. We

may use the transformation to the dimensionless variables $l = \xi_{nm} / \mu$, p,

 $\tau = \omega_{nm}t$, and finally get a dynamical system (when $\omega = 2\pi 2.27$ Hz and $\mu = 0.26$ m) in the following form $\dot{l} = p$ $\dot{p} = -l - \alpha p + 1.0504l^3 - 1.4003lp^2 - 0.0478Al \sin(2\tau - \beta\tau) + 0.0299Ap \cos(2\tau - \beta\tau)$ This system (at $\beta = 2 - \frac{\omega}{\omega_{nm}} = 0.014$ and additional damping forces with $\alpha = 0.01$) has for any initial conditions only regular solutions. As an

with $\alpha = 0.01$) has for any initial conditions only regular solutions. As an example in the fig.1 the phase portraits for different values of parameter A (which is proportional to the amplitude of wavemaker oscillations) are shown. Power spectra are presented in fig.2. They are discrete for different values of A.



Fig. 1. Phase portraits for different values of wavemaker oscillations A.





Now we theoretically consider the nonlinear problems of fluid free-surface waves which are excited by inner shell vibrations in a volume between two cylinders of finite length. It is useful to relate the fluid motion to the cylindrical coordinate system (r, θ, x) . The fluid has an average depth d; the average position of the free surface is taken as x = 0, so that the solid tank bottom is at x = -d. The fluid is confined between a solid outer cylinder at $r = R_2$ and a deformable inner cylinder (which acts as the wavemaker) at average radius $R_1 = r_1 + a_0(d)^{-1} \int_{-d}^0 \cos(\eta x) dx = r_1 + 2a_0 / \pi$. This inner cylinder vibrates harmonically in such a way that the position of the wall of the inner cylinder is $r = R_1 + \chi_1(x,t) = R_1 - (a_0 + a_1 \cos \omega t) \cos \eta x - 2a_0 / \pi$, where $\eta = \pi / (2d)$. The potential ϕ can be written as the sum of three harmonic functions $\phi = \phi_0 + \phi_1 + \phi_2$, Lamé, 1852, [8]. The solution of the linear problem for ϕ_1 can be written in the form, Krasnopolskaya, 1996, [4]

$$\phi_{1} = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} \phi_{ij}^{c,s}(t) \frac{\cosh k_{ij}(x+d)}{N_{ij}\cosh k_{ij}d} \psi_{ij}^{c,s}(r,\theta),$$
(3.1)

on the complete systems of azimuthal $(\cos i\theta, \sin i\theta)$, and radial eigenfunctions $\chi_{ij}(k_{ij}r) = J_i(k_{ij}r) - \frac{J_{i'}(k_{ij}R_1)}{Y_{i'}(k_{ij}R_1)}Y_i(k_{ij}r)$, with some arbitrary amplitudes $\phi_{ij}^{c,s}(t)$. In the solution (3.1) the notations $w_{ij}^{c,s}(r,\theta) = \chi_{ij}(k_{ij}r)\cos(\theta\sin(i\theta))$ are used where J_{ij} and V_{ij} are the *i*, the

 $\psi_{ij}^{c,s}(r,\theta) = \chi_{ij}(k_{ij}r)(\cos i\theta, \sin i\theta)$ are used, where J_i and Y_i are the *i*-th order Bessel functions of the first and the second kind, respectively, and N_{ij} is a normalization constant, where the index *c* (or *s*) indicates that the eigenfunction $\cos i\theta$ (or $\sin i\theta$) is chosen as the circumferential component; k_{ij} represents eigen wave numbers. The system of functions $\psi_{ij}(r,\theta)$, with i=0,1,2,... and j=1,2,3,..., is a complete orthogonal system, so any function of the variables *r* and θ can be represented using the usual procedure of Fourier series expansion. Thus, the free surface displacement $\zeta(r,\theta,t)-\zeta_0(t)$ can be written as $(\zeta_0(t))$ is the mean level of fluid free surface oscillations)

$$\zeta(r,\theta,t) - \zeta_0(t) = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} \zeta_{ij}^{c,s}(t) \frac{\psi_{ij}^{c,s}(r,\theta)}{N_{ij}}.$$
(3.2)

Under a parametric resonance, when the excitation frequency is twice as large as one of the eigenfrequencies, i.e. $\omega \approx 2\omega_{nm}$, and according the experimental

observations we may assume that the free-surface displacement can be approximated by two resonant modes. So that we may write [4]

$$\zeta \approx \frac{1}{N_{nm}} \zeta_{nm} \psi_{nm}^c(r,\theta) + \frac{1}{N_{0l}} \zeta_{0l} \psi_{0l}(r) + \zeta_0$$
(3.3)

where ψ_{0l} is the axisymmetric mode which has the eigenfrequency by a value very close to ω , i.e. $\omega_{0l} \approx \omega$. From the experimental observations follows that cross-waves has ampliteds much bigger than the amplitudes of the forced waves with the frequency ω of the wavemaker vibrations. So that we can seek the unknown functions in the form

$$\zeta_{nm}(t) = \varepsilon_1^{1/2} \lambda_1 \left[p_1(\tau_1) \cos \frac{\omega t}{2} + q_1(\tau_1) \sin \frac{\omega t}{2} \right];$$

$$\zeta_{0l}(t) = \varepsilon_1 \lambda_0 \left[p_2(\tau_1) \cos \omega t + q_2(\tau_1) \sin \omega t \right],$$
(3.4)

where
$$\lambda_1 = k_{nm}^{-1} \operatorname{th}(k_{nm}h)$$
, $\varepsilon_1 = \frac{a\omega_{nm}^2}{g}$ is a small parameter, $\tau_1 = \frac{1}{4}\varepsilon_1 \omega_1$

is a dimensionless slow time, $\lambda_0 = k_{0l}^{-1} \operatorname{th}(k_{0l}h)$. By substitution of the expressions (3.4) into boundary conditions, Krasnopolskaya, 1996, [4] and averaging over the fast time ωt we finally obtain the dynamical system in the form, Krasnopolskaya, 2013, [7],

$$\frac{dp_{1}}{d\tau_{1}} = -\alpha p_{1} - \vartheta q_{1} + \beta_{3}q_{1} + \beta(q_{1}p_{2} - p_{1}q_{2});$$

$$\frac{dq_{1}}{d\tau_{1}} = -\alpha q_{1} + \vartheta p_{1} + \beta_{3}p_{1} + \beta(p_{1}p_{2} + q_{1}q_{2});$$

$$\frac{dp_{2}}{d\tau_{1}} = -\alpha p_{2} - \beta_{2}q_{2} - 2\beta_{4}p_{1}q_{1};$$

$$\frac{dq_{2}}{d\tau_{1}} = -\alpha q_{2} + \beta_{2}p_{2} + \beta_{4}(p_{1}^{2} - q_{1}^{2}) + \beta_{5},$$
(3.5)

where $\mathcal{G} = \left[\beta_1 + \frac{\beta_6}{2}(p_1^2 + q_1^2)\right], \ \alpha = \frac{\delta}{\omega_{nm}}, \ \delta$ is the ratio of actual to

critical damping of the mode, β_i (*i*=1,2,...6) are constant coefficients. The dynamical system (3.5) is nonlinear, so numerical solutions were obtained. We

used the following coefficients (Krasnopolskaya, 1996, [4] – Becker, 1991, [1]) and data:

$$\alpha = 0.01; \ \beta_3 = 1.3k; \ \beta_4 = 0.25; \ \beta_5 = 0.235k; \ \beta_6 = 1.12; \ \beta = -1.531;$$

$$p_1(0) = q_1(0) = p_2(0) = q_2(0) = 0.5.$$

For these parameters and for different values of k (which is dimensionless amplitude of the wavemaker vibrations) extensive numerical calculations were carried out in order to find all steady state regimes. In Figure 3 dependences of the maximum Lyapunov exponents on value k are shown for the different values of the detuning parameters β_1 and β_2 .



Fig. 3. The dependence of the maximum Lyapunov exponent on value k.

Comparing these dependencies we may conclude that the dynamical system, which corresponds to the case when there is no detuning between the half of the frequency of excitation ω and the eigenfrequency of the cross-waves ω_{nm} , i.e. $\beta_1 = 0$, and there is the detuning of frequencies for the axisymmetric mode

 $\beta_2 = 0.2$, has chaotic regimes in the wider area of the parameter k changing. To demonstrate this we show in Figure 4 and 6 the phase portraits of solutions for the first case and the second when $\beta_1 = 0.2$ and there is no detuning for the axisymmetric mode, i.e. $\beta_2 = 0$. In Figure 4 c) we have the chaotic attractor and in Figure 6 c) the regular cycle. And in Figure 4 attractors occupy bigger areas. Power spectra for considered cases are show in Figures 5 and 7 correspondently.



when $\beta_1 = 0$, $\beta_2 = 0.2$.





Fig. 5. Power spectra computed for p_1 data (cases a, b, c and d) when $\beta_1 = 0$, $\beta_2 = 0.2$.



Fig.6. Phase portraits for regular (cases a, b) and chaotic regimes (cases c, d) when $\beta_1 = 0.2$, $\beta_2 = 0$.



Fig. 7. Power spectra computed for p_1 data (cases a, b, c and d) for the case

when $\beta_1 = 0.2$, $\beta_2 = 0$.

As we may conclude from numerical data and graphs in Figures 3-7 the dynamical system (3.5) has both regular and chaotic regimes. The chaotic regimes could be realized when $k \ge 1$ for the first case and $k \ge 1.6$ for the second considered case. For such values of corresponding amplitudes of wavemaker oscillations the largest Lyapunov exponents are positive, phase portraits have complicated structures of trajectory sets and power spectra are continuous ones.

4 Conclusions

Two new models expressing interaction of two eigenmodes at the condition of parametric resonances for the cross-waves of fluid free surface oscillations are developed. Models are simulated. The existence of chaotic attractors was established for the dynamical system presenting cross-waves and forced waves
interaction at fluid free-surface in a volume between two cylinders of finite length. For the system describing resonant cross-waves in the rectangular tank no chaotic regimes were found because the connection coefficients of crosswaves with the axisymmetric waves under the forced resonance are values on much smaller order then considered here. So that there are less factors to destabilize the system.

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Synthesis of Control Laws Sea Launch Aerospace System on the Basis of Super-heavy Amphibian Aircraft

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Abstract. Main task of modern airspace industry is delivery heavy-weight cargo element on earth orbit. Lunching spacecraft by aerospace system of "sea launch" nearby equator is preferably from economical point of view. Aerospace system which based on superheavy amphibious aircraft Be-2500 was offered as the subject of the study. The purpose of research is development of nonlinear algorithm of motion control of aerospace system on take-off and injection into Earth orbit. This algorithm provides desirable value of airspeed, orbital altitude and prevents withdrawal from flying path. There is used the synergetic method of analytical designing of aggregated regulators (ADAR) to synthesize the basic control laws.

Keywords: Aerospace System, Synthesis of Control Laws.

1 Introduction

Main task of modern airspace industry is delivery heavy-weight cargo element on earth orbit. Lunching spacecraft by aerospace system of "sea launch" nearby equator is preferably from economical point of view. Launch failure from the platform "Sea Launch" may cause a serious threat for ecology of World oceans. Therefore, the most appropriate for injection spacecraft into orbit is using of aerospace system "Air Start", based on project super heavy amphibious aircraft Be-2500, which developed at JSC "Beriev Aircraft Company. Air Start system consists of lunch aircraft, upper-stage rocket and spaceplane, see Figure 1 (Kobzev et al. [1]). All stages are returnable and reusable. Super-heavy amphibious aircraft can transport upper-stage rocket and spaceplane in any point on the planet and take-off as from the airport and from water. Injection into Earth orbit consists from three stages. On the first stage: take-off of launch aircraft with upper-stage rocket with spaceplane aboard and lift at release altitude of upper-stage rocket. Upper-stage rocket with spaceplane aboard are separating from launch aircraft. On the second stage: upper-stage rocket with spaceplane aboard reaching hypervelocity acceleration and spaceplane separating from upper-stage rocket. On the third stage: solo flight of spaceplane

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and placing in Earth orbit. Transporting of Super-heavy amphibious aircraft to any point on the planet increase mobility of space lunching complexes.

Fig. 1. Aerospace System: 1 – Lunch Aircraft; 2 – Upper-stage Rocket; 3 – Spaceplane

Using of horizontal component of the velocity vector instead vertical launch vehicle allow injecting into Earth orbit cargo element with larger mass. Special requirements are demanding for precision piloting aerospace system and for docking with the orbiting space station. Therefore, it is urgent to develop a system of automatic control of all stages of the Aerospace system.

2 Mathematical model of Aerospace system

Nonlinear mathematical model of aerospace system consists of mathematical models of lunch aircraft, upper-stage rocket and spaceplane. Dynamic equation takes into account following factors: aerodynamic configuration, type and location of engines, control device, cross-feed of aerospace system elements. Mathematical model of the spatial motion of lunch aircraft considering below (Byushgens and Studnev [2]):

$$\dot{V}_{x}(t) = V_{y}\omega_{z} - V_{z}\omega_{y} + \frac{1}{m}(-mg\sin\theta - X^{s} + \sum_{j=1}^{k}P_{xj});$$

$$\dot{V}_{y}(t) = V_{z}\omega_{x} - V_{x}\omega_{z} + \frac{1}{m}(-mg\cos\theta\cos\gamma + Y^{s} + \sum_{j=1}^{k}P_{yj});$$

$$\dot{V}_{z}(t) = V_{x}\omega_{y} - V_{y}\omega_{x} + \frac{1}{m}(mg\cos\theta\sin\gamma + Z^{s} + \sum_{j=1}^{k}P_{zj});$$

(1)

$$\begin{split} \dot{\omega}_{x}(t) &= \frac{I_{y} - I_{z}}{I_{x}} \omega_{y} \omega_{z} + \frac{1}{I_{x}} (M_{x}^{s} + \sum_{j=1}^{k} M_{xpj}); \\ \dot{\omega}_{y}(t) &= \frac{I_{z} - I_{x}}{I_{y}} \omega_{x} \omega_{z} + \frac{1}{I_{y}} (M_{y}^{s} + \sum_{j=1}^{k} M_{ypj}); \\ \dot{\omega}_{z}(t) &= \frac{I_{x} - I_{y}}{I_{z}} \omega_{x} \omega_{y} + \frac{1}{I_{z}} (M_{z}^{s} + \sum_{j=1}^{k} M_{zpj}); \\ \dot{x}(t) &= V_{x} \cos \psi \cos \vartheta + V_{y} (\sin \gamma \sin \psi - \cos \gamma \cos \psi \sin \vartheta) + \\ + V_{z} (\cos \gamma \sin \psi + \sin \gamma \cos \psi \sin \vartheta); \\ \dot{y}(t) &= V_{x} \sin \vartheta + V_{y} \cos \gamma \cos \vartheta - V_{z} \sin \gamma \cos \vartheta; \\ \dot{z}(t) &= -V_{x} \sin \psi \cos \vartheta + V_{y} (\sin \gamma \cos \psi + \cos \gamma \sin \psi \sin \vartheta) + \\ + V_{z} (\cos \gamma \cos \psi - \sin \gamma \sin \psi \sin \vartheta); \\ \dot{\vartheta} &= \omega_{y} \sin(\gamma) + \omega_{z} \cos(\gamma); \\ \dot{\gamma} &= \omega_{x} - tg(\vartheta)(\omega_{y} \cos(\gamma) - \omega_{z} \sin(\gamma)); \\ \dot{\psi} &= \frac{1}{\cos(\vartheta)} (\omega_{y} \cos(\gamma) - \omega_{z} \sin(\gamma)), \end{split}$$

where $V_x, V_y, V_z, \omega_x, \omega_y, \omega_z$ - projections of linear and angular speed on coupled coordinate system; x, y, z - coordinates of the center of mass of lift aircraft in the earth coordinate system; $\mathcal{G}, \gamma, \psi$ - pitch attitude, bank attitude, yaw attitude; I_x, I_y, I_z - lift aircraft moment of inertia; g - acceleration of gravity; m - aerospace system mass;

$$\begin{split} X^{s} &= X + \Delta X_{RB+BKC} + \sum_{i=1}^{n} N_{xi} ; \qquad M_{x}^{s} = M_{xa} + \Delta M_{xRB+BKC} + \sum_{i=1}^{n} M_{xNi} ; \\ Y^{s} &= Y + \Delta Y_{RB+BKC} - \sum_{i=1}^{n} N_{yi} ; \qquad M_{y}^{s} = M_{ya} + \Delta M_{yRB+BKC} + \sum_{i=1}^{n} M_{yNi} ; \\ Z^{s} &= Z + \Delta Z_{RB+BKC} + \sum_{i=1}^{n} N_{zi} ; \qquad M_{z}^{s} = M_{za} + \Delta M_{zRB+BKC} + \sum_{i=1}^{n} M_{zNi} ; \end{split}$$

 $\Delta X_{RB+BKC}, \Delta Y_{RB+BKC}, \Delta Z_{RB+BKC} - \text{ increment of drag force, normal aerodynamic force and sideway force from upper-stage rocket with spaceplane;}$ $<math display="block">\sum_{i=1}^{n} N_{xi}, \sum_{i=1}^{n} N_{yi}, \sum_{i=1}^{n} N_{zi} - \text{projection of forces of upper-stage rocket- lift aircraft retention mechanism in coupled coordinate system;$ *i* $- index number of retention mechanism support column; <math display="block">\sum_{j=1}^{k} P_{xj}, \sum_{j=1}^{k} P_{yj}, \sum_{j=1}^{k} P_{zj} - \text{projection of summary force of main engines and lift engines on axis of coupled coordinate$ system; *j* - number of engine; $\Delta M_{xRB+BKC}$, $\Delta M_{yRB+BKC}$, $\Delta M_{zRB+BKC}$ - moment of aerodynamic force from upper-stage rocket with spaceplane about the axis of coupled coordinate system with the origin at the lunch aircraft center of mass; $\sum_{i=1}^{n} M_{xNi}, \sum_{i=1}^{n} M_{yNi}, \sum_{i=1}^{n} M_{zNi}, - \text{moment of force } \sum_{i=1}^{n} N_{xi}, \sum_{i=1}^{n} N_{yi}, \sum_{i=1}^{n} N_{zi} \text{ about}$

the axis of coupled coordinate system; $\sum_{j=1}^{k} M_{xPj}$, $\sum_{j=1}^{k} M_{yPj}$, $\sum_{j=1}^{k} M_{zPj}$ - moments

of the engine thrust about the axis of coupled coordinate system.

3 Synthesis of adaptive control laws

A distinctive feature at the initial stage of take-off of aerospace system, based on super-heavy amphibious aircraft Be-2500, is creating under the center wing section dynamic air cushion from combined work of main engines and lift engines (Kobzev *et al.* [1]). Acceleration of lift aircraft provided by thrust of main engines and horizontal component of lift engines. With increasing ascentional force, aircraft completely separated from the water and lift-off. Climb and primary mission carried out over the desert areas, which minimizes the potential pollution of the ocean in an accident (Kobzev *et al.* [1]).

This report proposes the synergetic method of analytical designing of aggregated regulators (ADAR) (Kolesnikov [3]) to the problem of designing control strategies of an aerospace system. At the initial stage of lunch aircraft takeoff, where the lunch aircraft had already lifted the starting surface, it is necessary to stabilize the lunch aircraft and orientate on a course for further forward movement. Aircraft speed is still insufficient for the effective operation of the aerodynamic controls (elevator, rudder, aileron, etc.), and control of aircraft is realized by varying of thrust of the lift and main engines. Synthesize vector control law u(t), capable to providing the predetermined invariants (objectives of management) $V_x = V_x^* = 0$, $V_y = V_y^* = 0$, $V_z = V_z^* = 0$,

 $\mathcal{G} = \mathcal{G}^* = 0$, $\psi = \psi^*$ and able to parry external unmeasured disturbance (Bukov [4], Podchukaev [5]). In accordance with the method ADAR, write extended mathematical model of synergistic synthesis in variables of system status obtained from the model (1) by adding integrators to the original system:

$$\dot{x}_{1}(t) = x_{2}x_{6} - x_{3}x_{5} - g\sin x_{10} + \frac{1}{m}(X^{s} + u_{1} + z_{1});$$

$$\dot{x}_{2}(t) = x_{3}x_{4} - x_{1}x_{6} - g\cos x_{11}\cos x_{10} + \frac{1}{m}(Y^{s} + u_{2} + z_{2});$$

$$\dot{x}_{3}(t) = -x_{2}x_{4} + x_{1}x_{5} - g\sin x_{11}\cos x_{10} + \frac{1}{m}(Z^{s} + z_{3});$$

(2)

$$\begin{aligned} \dot{x}_{4}(t) &= \frac{-(J_{z} - J_{y})x_{5}x_{6} + M_{x}^{s} + u_{3}}{J_{x}}; \\ \dot{x}_{5}(t) &= \frac{-(J_{x} - J_{z})x_{4}x_{6} + M_{y}^{s} + u_{4}}{J_{y}}; \\ \dot{x}_{6}(t) &= \frac{-(J_{y} - J_{x})x_{4}x_{5} + M_{z}^{s} + u_{5}}{J_{z}}; \\ \dot{x}_{7}(t) &= x_{1}\cos x_{12}\cos x_{10} + x_{2}(\sin x_{11}\sin x_{12} - \cos x_{11}\cos x_{12} \cdot \cdots \cdot \sin x_{10}) + x_{3}(\cos x_{11}\sin x_{12} + \sin x_{11}\cos x_{12}\sin x_{10}); \\ \dot{x}_{8}(t) &= x_{1}\sin x_{10} + x_{2}\cos x_{11}\cos x_{10} - x_{3}\sin x_{11}\cos x_{10}; \\ \dot{x}_{9}(t) &= -x_{1}\sin x_{12}\cos x_{10} + x_{2}(\sin x_{11}\cos x_{12} + \cos x_{11}\sin x_{12} \cdot \cdots \cdot \sin x_{10}) + x_{3}(\cos x_{11}\cos x_{12} - \sin x_{11}\sin x_{12}\sin x_{10}); \\ \dot{x}_{10}(t) &= x_{5}\sin x_{11} + x_{6}\cos x_{11}; \\ \dot{x}_{11}(t) &= x_{4} - tg x_{10}(x_{5}\cos x_{11} - x_{6}\sin x_{11}); \\ \dot{x}_{12}(t) &= x_{5}\frac{\cos x_{11}}{\cos x_{10}} - x_{6}\frac{\sin x_{11}}{\cos x_{10}}; \\ \dot{z}_{1}(t) &= k_{1}x_{1}; \\ \dot{z}_{2}(t) &= k_{2}x_{2}; \\ \dot{z}_{3}(t) &= k_{3}x_{3}, \\ \text{where } x_{1} = V_{x}, \ x_{2} = V_{x}, \ x_{3} = V_{z}, \ x_{4} = \omega_{x}, \ x_{5} = \omega_{y}, \ x_{6} = \omega_{z}, \ x_{7} = x, \\ x_{8} = y, \ x_{9} = z, \ x_{10} = \vartheta, \ x_{11} = \gamma, \ x_{12} = \psi - \text{variables of system status}; \end{aligned}$$

$$u_{1} = \sum_{j=1}^{k} P_{xj}, \quad u_{2} = \sum_{j=1}^{k} P_{yj}, \quad u_{3} = \sum_{j=1}^{k} M_{xPj}, \quad u_{4} = \sum_{j=1}^{k} M_{yPj}, \quad u_{5} = \sum_{j=1}^{k} M_{zPj} - \sum_{j=1}^{k} M_{yPj}$$

controls; z_1 , z_2 , z_3 - dynamic variables, representing the external evaluation of unmeasured disturbances, acting on the control object; k_1 , k_2 , $k_3 > 0$ – fixed coefficients.

According to the method ADAR (Kolesnikov [3]), write the first array of macro variable: *

$$\psi_{1} = x_{1} - x_{1}^{*};$$

$$\psi_{2} = x_{2} - x_{2}^{*};$$

$$\psi_{3} = x_{4} - \phi_{1};$$

$$\psi_{4} = x_{5} - \phi_{2};$$

$$\psi_{5} = x_{6} - \phi_{3},$$
(3)

which must satisfy the system of functional equations

$$T_m \dot{\psi}_m(t) + \psi_m(t) = 0, \ m = 1, 2, ..., 5,$$
(4)

where: T_m – time constants, which affects to the quality of the dynamics of the process in a closed system " control object – regulator"; »; ϕ_l , l = 1,2,3 – any connecting function, so called "internal" control actions, which are selected in the subsequent stages of the synthesis procedure; ; $T_m > 0$ – condition of asymptotic stability in the whole equation (4) concerning to varieties $\psi_m = 0$. Putting in (4) at the intersection of invariant variety, obtain the system of algebraic equations:

$$x_1 = 0; \quad x_2 = 0; \quad x_4 - \phi_1 = 0; \quad x_5 - \phi_2 = 0; \quad x_6 - \phi_3 = 0.$$
 (5)

Express from the system of equations (5) "internal" controls ϕ_l and substitute them into the right sides equation of the object (2), where missing controls u_m . As a result, at the intersection of invariant variety $\psi_m = 0$ observed effect of dynamic "compression phase space" (Kolesnikov and Kobzev [6]). Dimension of the representative point of the system (2) decreases, the equation of decomposed system will assume an aspect:

$$\begin{aligned} \dot{x}_{3}(t) &= -g \sin x_{11} \cos x_{10} + (1/m) Z^{s} + (1/m) z_{3}; \\ \dot{x}_{7}(t) &= x_{3} \left(\cos x_{11} \sin x_{12} + \sin x_{11} \cos x_{12} \sin x_{10} \right); \\ \dot{x}_{8}(t) &= -x_{3} \sin x_{11} \cos x_{10}; \\ \dot{x}_{9}(t) &= x_{3} \left(\cos x_{11} \cos x_{12} - \sin x_{11} \sin x_{12} \sin x_{10} \right); \\ \dot{x}_{10}(t) &= \phi_{2} \sin x_{11} + \phi_{3} \cos x_{11}; \\ \dot{x}_{10}(t) &= \phi_{1} - tg x_{10} \left(\phi_{2} \cos x_{11} - \phi_{3} \sin x_{11} \right); \\ \dot{x}_{12}(t) &= \phi_{2} \frac{\cos x_{11}}{\cos x_{10}} - \phi_{3} \frac{\sin x_{11}}{\cos x_{10}}; \\ \dot{z}_{1}(t) &= k_{1}x_{1}; \\ \dot{z}_{2}(t) &= k_{2}x_{2}; \\ \dot{z}_{3}(t) &= k_{3}x_{3}, \end{aligned}$$

For decomposed system (6) introduces a second set of macro variables:

$$\psi_{6} = x_{10} - x_{10}^{*};$$

$$\psi_{7} = g \sin x_{11} \cos x_{10} + (1/m)Z^{s} + (1/m)k_{5}z_{3} + k_{4}x_{3};$$

$$\psi_{8} = x_{12} - x_{12}^{*},$$
(7)

where: $k_4, k_5 > 0$ – fixed coefficients.

Set of macro variables (7) must satisfy the solution of the system of functional equations:

$$T_h \dot{\psi}_h(t) + \psi_h(t) = 0, \ h = 6,7,8,$$
(8)

where: T_h – time constants, which affects to the quality of the dynamics of the process in a closed system. Combined analytical solution of equations (6), (7) and (8) allow to find an expression for the 'internal' controls ϕ_l , as functions variables of the system status $x_3, x_{10}, x_{11}, x_{12}$, time constants T_h , dynamic variable z_3 and technologic invariant x_{12}^* . Solving the system of functional equations:

$$T_{6}\dot{\psi}_{6} + \psi_{6} = 0;$$

$$T_{7}\dot{\psi}_{7} + \psi_{7} = 0;$$

$$T_{8}\dot{\psi}_{8} + \psi_{8} = 0,$$
(9)

Obtain expressions for the "internal" controls:

$$\begin{split} \phi_{1} &= -(T_{6}T_{7}g\cos(x_{11})\cos(x_{10})^{2}tg(x_{10})(x_{12} - x_{12}^{*}) + T_{6}T_{8}(T_{7}k_{4} + \\ &+ mk_{5} + 1)z_{3} + T_{6}T_{8}(k_{4} + 2T_{7}k_{3})x_{3} + (T_{6}T_{7}T_{8}k_{4} + T_{6}T_{8} + T_{7}T_{8}x_{10}) \cdot \\ \cdot g\sin(x_{11})\cos(x_{10}))/(T_{6}T_{7}T_{8}g(\cos(x_{11})\cos(x_{10})); \\ \phi_{2} &= \phi_{3}\text{ctg}(x_{11}) + \frac{1}{T_{6}\sin(x_{11})}x_{10}; \\ \phi_{3} &= \frac{-T_{8}\cos(x_{11})x_{10} + T_{6}\sin(x_{11})\cos(x_{10})(x_{12} - x_{12}^{*})}{T_{6}T_{8}(\cos(x_{11})^{2} + \sin(x_{11})^{2})} \end{split}$$
(10)

Solving the system (4) of functional equations of ADAR method together with the expressions obtained for ϕ_l (10), equations of the model object (2) and macro variables (3), obtain the desired external controls as a function depending on the variables of system status:

$$u_{1} = f(x_{1}, x_{2}, x_{3}, x_{5}, x_{6}, x_{10}, z_{1});$$

$$u_{2} = f(x_{1}, x_{2}, x_{3}, x_{4}, x_{6}, x_{10}, x_{11}, z_{2});$$

$$u_{3} = f(x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}, x_{10}, x_{11}, x_{12}, z_{3});$$

$$u_{4} = f(x_{4}, x_{5}, x_{6}, x_{10}, x_{11}, x_{12});$$

$$u_{5} = f(x_{4}, x_{5}, x_{6}, x_{10}, x_{11}, x_{12}).$$
(11)

4 Modeling

Substituting the expressions obtained for the control laws (11) in the extended model of the object (2), setting the controller parameters and technological invariants, obtain a closed system "object - a regulator." In the right sides of the controlled variables introduce piecewise constant disturbance instead of estimates of perturbations z_1, z_2, z_3 . Results of numerical modeling with parameters of a specific object controls are presented at figure 2÷11.





Conclusions

Results of simulation show, that the obtained control laws provide asymptotic stability of the closed nonlinear system, realize objectives of management and compensate external disturbances, therethrough reduce crew workload and enhance the safety of the lunch aircraft at the initial phase of takeoff.

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Computer modeling of information properties of deterministic chaos

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Abstract. Since mathematical models describing the work of transmitting-receiving units of modern chaotic information systems have become more complex, modeling of information properties of deterministic chaos is becoming more topical. The paper presents the results of a wide range of works related to the modeling of dynamic chaos usage in modern telecommunication systems - from generating chaotic sequences to their application for information security and as the actual information media.

Keywords: Deterministic chaos, chaotic system, computer modeling, information properties.

1 Introduction

Nowadays, there is a rapid development of both new methods of information transmission and security, and new means of processing analog and digital information flows that come in opened or closed state. Deterministic chaos is one of the new elements which is recently started to be frequently used in modern communication systems Banerjee et al.[1]. In short, this phenomenon is complex nonperiodic oscillations that occur under certain conditions (parameters) and are the inner nature of the so-called chaotic dynamical systems Cvitanovic et al.[2]. This paper presents generalized complex results on the dynamic chaos usage in modern communication systems, which are carried out in the laboratory for the study of chaotic processes in radio-engineering of the Physical, Technical and Computer Science Institute of Chernivtsi National University.

The paper has the following structure. In the second section the relation between the Lyapunov exponents and information properties of chaotic oscillations is analyzed. The third section is devoted to the modeling of hyperchaotic systems in the environment LabView. In the fourth section some topical issues of the dynamic chaos usage for information security are discussed, namely the formation of pseudorandom generators based on two chaotic systems. The fifth section presents the studies of models of information systems using deterministic chaos and also some obtained numerical characteristics.

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2 Lyapunov exponents and information properties of chaotic signals

For practical use of chaotic signals it is necessary to use criteria of signals complexity. The characteristics of chaotic signals, allowing them to be compared include: fractal dimensions (correlation dimension, information dimension), Fourier spectrum, Poincare section, Lyapunov exponents, topological entropy, etc Francis C. Moon[3]. Fractal dimension of the attractor allow to evaluate the metric complexity of its trajectories in phase space. Fractal characteristics of chaotic attractors are invariant to the time scale of chaotic systems. The information properties of signals are important for communication, cryptography and other applications. Visual image of a dynamic system is its attractor.

Informational properties of chaotic oscillations can be estimated using the Lyapunov exponents. In the theory of dynamical systems the Lyapunov exponent is a quantitative measure of the exponential divergence of initially close trajectories. If the initial distance between the trajectories is d_0 then at time t the average distance between them will be $d \langle \cdot = d_0 e^{\lambda t},$ where $\lambda -$ Lyapunov exponent. In terms of information theory, the largest Lyapunov exponent is numerically equal to the average information created by a dynamic system. Next, we show that the Lyapunov exponents are dependent on the time scale of the dynamic system.

Consider the Rossler system, described by the system of three differential equations Rossler[4]:

$$\begin{cases} \frac{dx}{dt} = -(y+z), \\ \frac{dy}{dt} = x - ay, \\ \frac{dz}{dt} = b + z(x-c), \end{cases}$$
(2.1)

where x, y, z – state variables, a = 0.15, b = 0.2, c = 10 – system parameters for which there is a chaotic regime.

The values of the Lyapunov exponents of the system (2.1) are as follows: $\lambda_1 = 0.09$, $\lambda_2 = \lambda_3 = -9.82$. We will change time scale in the system (2.1) by replacing $t = kt \square$, where k > 0, and obtain a system (2.2):

$$\begin{cases} \frac{dx}{dt} = -k(y+z), \\ \frac{dy}{dt} = k(x-ay), \\ \frac{dz}{dt} = k(b+z(x-c)), \end{cases}$$
(2.2)

Systems (2.1) and (2.2) have the same chaotic attractors and fractal dimensions, but the Lyapunov exponents of the system (2.2) are linearly dependent on the parameter k as shown in Figure 2.1.

By varying the time scale of chaotic systems (2.2) by changing the parameter k it is possible to control the speed of generating information. This is a practical method of information properties management of dynamical systems. At the same time, the change of time scale of chaotic systems is equivalent to the change of width of the oscillations spectrum in k times. This means that the value of the senior Lyapunov exponent and the width of the signal spectrum are interconnected. For example, consider two chaotic flow systems – the Rossler system (2.1) and the Lorenz system Lorenz[5]:

$$\begin{cases} \dot{x} = \sigma(y-a), \\ \dot{y} = rx - y - xz, \\ \dot{z} = xy - bz, \end{cases}$$
(2.3)

where $\sigma = 10$, r = 28, b = 8/3 – system parameters.



For the Lorenz system for the given parameters the values of the Lyapunov exponents are as follow: $\lambda_1 = -9.82$, $\lambda_2 = 0.9$, $\lambda_3 = -14.57$. The value of the largest Lyapunov exponent of the Lorenz system is greater than the largest Lyapunov exponent of the Rossler system in 10 times. As shown in Figure 2.2 and Figure 2.3 the signal spectrum of the Lorenz system is more complex and broader than in Rossler system.



In general, the signal complexity is a composit concept and includes spectral, information and metric data. Therefore, we can conclude that the Lyapunov exponent characterizes the signal complexity in terms of its information properties, but contains little information regarding the complexity of the metric structure of the signal. This means that it is incorrect to compare in general the signals complexity of continuous dynamic systems using only the value of the largest Lyapunov exponent.

3 Modeling of information properties of the hyper-chaotic Lorenz system

Hyper-chaotic Lorenz system is described by equations:

$$\begin{cases} \dot{x} = a(y-x), \\ \dot{y} = bx + y - xz - w, \\ \dot{z} = xy - cz, \\ \dot{w} = kyz, \end{cases}$$
(3.1)

where a,b,c – system parameters, x, y, z – initial conditions, k – constant that determines the attractor, which in some senses can be chaotic, and in particular – controlled Tiegang Gao et al.[6].

For modeling of information properties of the hyper-chaotic Lorenz system we used LabView programming environment [7].

Figure 3.1 shows the block scheme that implements of hyper-chaotic Lorenz system. The main functional part is a formula node, in which would include the equation (3.1). In the input formula node fed values of system parameters (a,b,c) and the value of the initial conditions (x, y, z). At the output assigned equations (3.1). Also, the output is an opportunity to demonstrate the solution of equations in three dimensions.

When changing the system parameters and initial conditions we can be analyzed in detail and investigate the behavior of a hyper-chaotic Lorenz system, which in many cases is a basic element of the functional blocks of chaotic secure communication systems.



Fig. 3.1. Block scheme of hyper-chaotic Lorenz system

Figure 3.2 shows the software interface which shows these information modeling properties as temporal distributions of the values of the coordinates X, Y, Z, three-dimensional map of hyper-chaotic attractor and phase portraits in the planes XY, XZ, i YZ, when the number of iterations N = 5000, the system parameters a = 10, b = 28, c = 8/3, k = 0,1, and initial conditions x = y = z = 1.



Fig. 3.2. Software interface which shows modeling of information properties

Figure 3.3 shows the spectral analysis of chaotic coordinates X, Y, Z with the number of iterations N = 5000 which was conducted using fast Fourier transform. The value 0.01 corresponds to 100 Hz.



Fig. 3.3. Fourier spectral analysis when the number of iterations N = 5000

Developed block diagram in LabView programming environment allows the program to explore the hyper-chaotic Lorenz system.

4 The use of dynamic chaotic systems in cryptography

Since with the development of information technology there is as well the development of means of data interception, there is a need in the development of new algorithms of information encryption. Dynamic systems are sensitive to the initial conditions and control parameters, which makes them good candidates for use in the development of encryption algorithms.

We have proposed a method of generating pseudorandom sequence of bits using two dynamic chaotic systems and operations XOR Shahtarin et al.[8]. The first dynamic system – the Lorenz system described by equation (4.1), the second – a logistic mapping described by equation (4.2).

$$\begin{cases} \dot{x} = -a(x - y), \\ \dot{y} = cx - y - xz, \\ \dot{z} = bz + xy, \end{cases}$$
(4.1)

where x, y, z – dynamic variables; a, b, c – parameters of the Lorenz system, which usually possess the values a = 10, b = 8/3, c = 28.

$$\upsilon_{n+1} = r\upsilon_n(1-\upsilon_n), \qquad (4.2)$$

where v_n and r – system variable and system parameter respectively, n – iteration number. The system parameter r is a significant part of the equation and if the values 3.57 < r < 4 the system is characterized by chaotic behavior.

Both dynamic systems were used to generate values of dynamic variables Arvind et al.[9]. The values of dynamic variables x, y and z of the Lorenz system were compared with the generated value v of logistic mapping. If the value of Lorenz system variable was larger than the value of the variable of logistic mapping, a decision was made that the generated logical «1» otherwise logical «0». Thus three sequences of bits are being generated k_1 , k_2 and k_3 that are joined together using XOR operation thus forming total pseudorandom sequence of bits. Then the obtained sequence can be used for information encryption.

However, for the correct operation of such generator it is necessary to coordinate the range of output values of the Lorenz system with the range of output values of logistic mapping. This is done by mapping the obtained value of the variable within the interval (0;1).

We have performed simulation of the proposed generator operation in the environment LabView, block diagram of the generator is shown in Figure 4.1. Simulation has shown that the proposed generator can be easily implemented by software and is quite quick Kosovan[10].



Fig. 4.1. Block diagram of the generator based on two dynamic systems

Also, we have implemented the proposed generator in the programming language Delphi 7 an external view of the program is shown in Figure 4.2.

🌽 Form1		. 🗆 🛛					
×0= 0,1347	a= 10						
y0= 0,9573	b= 2.67						
z0= 0,3681	c= 28						
v0= 0,1562	r= 3,99819	close					
dt= 0,05741							
n 16000000							
згенерувати та зберегти							



To check whether the proposed generator has the properties of a pseudo randomness, the sequence of bits with the length of 16000000 bits was generated. The sequence generation was performed when the values of initial conditions were the following: for the Lorenz system $x_0 = 0.1347$, $y_0 = 0.9573$, $z_0 = 0.3681$, a = 10, b = 2.67, c = 28 and integration step dt = 0.05741 for cubic mapping $v_0 = 0.1562$ and r = 3.9979. In this algorithm the integration step dt and control parameter r play a significant role in keys forming, so it is necessary to carefully select their values to be able to obtain the generated pseudorandom sequence of bits of large length.

The obtained sequence was tested using a set of statistical tests NIST STS-1.6. 15 of 16 tests passed.

On the basis of obtained results we can conclude that the generated sequence is really pseudorandom and the proposed generator can be used in the development of algorithms of information encryption. Also the proposed generator has a large number of keys (initial conditions and parameters), namely 9 of which 6 can change their values over the sufficiently wide range. If you set keys with an accuracy of 5 decimal places a number of their possible combinations will be approximately 10^{35} . Such a large number of keys complicates their selection and makes brute-force attack more complex and costly.

5 The research of the possibility of information recovery, its hiding and noise immunity in information systems using deterministic chaos

Nowadays, the development of digital systems of hidden communication using chaotic signals is a topical issue. Numerous works offer analog communication systems that use the synchronization of transmitter and receiver for data recovery Politansky et al., Eliyashiv et al.[11-12]. The research has found that such systems possess low noise immunity caused by high sensitivity of chaotic synchronization to the noises in the communication channel and by the parameters detuning of drive and response generators. The use of digital systems provides both the rise of noise immunity level of data transmission process, compared to the analog ones, and the possibility of encoding Bollt, Lai [13] and cryptographic security methods Baptista[14] application.

The most widely used scheme for hidden digital communication is the chaotic switching scheme using full synchronization phenomenon Koronovskii[15]. Among the systems of hidden transmission of analog information the most widely used is the circuit with the use of chaotic masking Downes, Ivanyuk et al.[16-17] that is analytically defined by the system of differential equations (5.1). The principal of system operation is as follows. One of the output chaotic oscillations of the generator x(t) is summed up with an analog data signal m(t) followed by transmission to channel. Security of the data transmission process through the channel is ensured by complete overlap of the data signal spectrum by chaotic oscillation spectrum. The receiver contains one chaotic generator

u(t), identical to transmitter generator. Recovered signal can be obtained after passing through subtractor as the difference between the receiver input signal and the response generator output signal. The control parameters variety of drive and response generators and the presence of noises in communication channel results in arising of synchronization error that equals the error of data signal recovery. Desynchronization of transmitter and receiver generators eliminates the possibility of data recovery, transmitted through the channel. Besides, it is necessary to ensure the ratio signal/noise no less than 35 dB for accurate data recovery, which is its principal disadvantage Vovchuk et al.[18].

$$\begin{cases} \dot{x} = \alpha(y - x - f(x)), \\ \dot{y} = x - y + z, \\ \dot{z} = -\beta y, \end{cases} \qquad \begin{cases} \dot{u} = \alpha(v - u - f(v)) + e(x - u), \\ \dot{v} = x - v + w, \\ \dot{w} = -\beta v, \end{cases}$$
(5.1)

where x, y, z, u, v, w – dynamic variables; e – coupling coefficient.

When using the circuit of chaotic masking in digital systems of data transmission, the hiding of transmission process through the channel will be low as in the intervals, that are equal to the duration of data bit transmission, a strong constant component will take place. In order to eliminate this defect we offer a modification of analog circuit of chaotic masking for digital communication [18]. In contrast to the circuits of analog data transmission it contains a subsidiary generator G, the signal of which is modulated by digital data signal and added to the chaotic signal. The modulation is carried out with the aid of a key which is turned on or off depending on the value of data bit. The implementation of preliminary modulation and ensuring the identity of statistic and spectral characteristics of signals generated by the generator G and the masking oscillation x(t) enables to match the parameters of carrying and chaotic signals. Both harmonic and chaotic signal can be used as a signal G(t) [18]. The receiver model remained unchanged. Mathematic model differ only by the presence another component in the fifth equation that describes the type of modulated carrier oscillation, namely $m(t)Asin(2\pi ft)$ or m(t)y(t), when using the harmonic or chaotic oscillation, respectively.

If the chaotic oscillation is used as carrier then it is sufficient for hidden communication that its spectrum is completely offset by masking oscillation one. There is other situation using harmonic signal as carrier. In this case, the hiding in the channel depends on its frequency and amplitude values.

The harmonic signal hiding decreases with increasing the value of its amplitude. But the decrease in harmonic signal amplitude leads to the decrease in power of desynchronization signal of drive and response systems and consequently to the decrease in noise immunity of information transmission in general. Thus, for reliable operation of the system with chaotic masking it is necessary to choose a compromise between the chaotic and harmonic signal values.

In the modeling process the amplitude A and frequency f were varied. The

curves family (Figure 5.1) of a dependence $\frac{P_{desyn}}{P_{ms}}(\frac{P_{hs}}{P_{ms}})$, where

 $P_{desyn} = P_{S(t)} - P_{u(t)}$ - power of a desynchronization signal, $P_{S(t)}$ - signal power

in the channel, $P_{u(t)}$ - power of response generator output signal, P_{hs} - harmonic signal power, P_{ms} - masking signal power. The figure analysis showed that increasing of harmonic signal amplitude leads to increase in value P_{desyn} . The dependence is linear when the values f are up to 1 kHz, while P_{desyn} does not exceed 20 % of P_{ms} . An increase f leads to the complication of dependence. When f are increasing closer to the upper frequency spectrum of a chaotic signal $f_7 = 3,2$ kHz ta $\frac{P_{hs}}{P_{ms}} > 0.04$, the value P_{desyn} practically does not depend on A and has 80-90 % of P_{ms} . If f goes beyond the chaotic oscillation spectrum, the dependence $\frac{P_{desyn}}{P_{ms}}(\frac{P_{hs}}{P_{ms}})$ gets more complicated and even when P_{ms}



Fig. 5.1. The dependence of the normalized power of the desynchronization signal on the normalized power of the harmonic signal by changing the values of the amplitude and frequency of the harmonic signal



Fig. 5.2. Dependence of the probability of incorrect bits recovery on the value of signal/noise ratio in communication channel (1 – with harmonic oscillation used as a carrier signal; 2 –chaotic switching scheme; 3 – with chaotic oscillation used as a carrier signal)

Therefore, for the improvement of quality of information recovery it is reasonable to use the harmonic signal with a frequency close to the upper frequency of the chaotic signal. The obtained results can also be used for analog communication systems, where the harmonic oscillation is the information.

The dependence of error probability of the received data on the value of signal/noise ratio in communication channel is shown in Figure 5.2. The obtained results show that the system of data transmission based on the usage of harmonic oscillation as a carrier signal yields to the chaotic switching scheme by its noise immunity (Figure 5.2 - curve 1 and curve 2 respectively).

The system based on the usage of chaotic oscillation as a carrier signal is more resistant to noise impact in the channel (curve 3). The error probability of recovery when using the modified circuit with the ratio S/N_0 of the order 10 dB is 10^{-3} , whereas its value constitutes 10^{-2} while using the chaotic switching scheme.

Conclusions

The results given in this paper once again demonstrate the importance of the extensive use of deterministic chaos in modern secure communication systems - both as a basic component for information encryption and encoding and as the actual information carrier. Since the behavior of information systems models is being studied in the various software environments, then on our opinion the special attention in future researches should be focused on the analysis of pseudorandom properties of chaotic sequences and the ability to control the behavior of chaotic systems. Speaking about the choice of one or another software environment, we would advise to pay attention to the system LabView, which makes it possible to analyze both software and hardware solutions in a very wide circuit range (from analog circuits to FPGAs).

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Acoustic decoding of a sheep bells and trotters within a hired of sheep

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Abstract: *Time series analysis is used to de-convolve bell and trotter signals within a hired of sheep for the purpose of identifying the sheep's activity: walking to and from grazing pasture and stock pens.*

Keywords: sheep bells, acoustic recoding, time series, overtones and walking gait.

1. Introduction

For centuries percussion instruments in the form of iron bells have been placed around the neck of sheep (also cattle and goats) to let herders know what's going on with the herd while they are doing other things. Indeed to improve awareness of the shepherds to the herd's activity, the loudest bell is placed on the more active bucks. The tranquil melodic bell ring while sheep are grazing has been used to locate herds on pasture, as well as letting the shepherds know that 'all is well'. In contrast the more rapid and louder ring tones have proven to be a good indicator of nearby predators. To the shepherd who has been broughtup to identify these two extremes, the acoustic signatures are easily identified, however it may be argued that the identifying conditions between these two extremes is much more difficult, and for the average person who is not involved in shepherding.

Time series analysis of a series values sampled of regular intervals has been shown to be a power tool in de-convolving complex noise sources such as: turbulent fluid [1], complex information with industrial plants chemical [2], and White Dwarf stars [3]. This paper reports upon the use of time series analysis and mathematical modelling of the acoustic response of iron bells that are attached to three sheep within a healthy herd containing between 30 healthy adult male and female 'Sfakia' sheep. The aim of this work is to establish if the bell acoustics can be used to identify the sheep's walking gait as they move steadily between pastures, rather than the extreme scenarios of grazing and threat of predators. To prevent imparted stress and 'sheep worrying' to a single sheep the acoustic measurements were performed at a distance of 20 m from the herd. The measurements were made in the month of October on the outskirts of the Cretan village of Kástelos in Western Crete. The time of the measurements are both in the morning (8-9 am local time) when the sheep are moved downhill to pastures and in the evening when they are moved back up-hill to the safety of their stock pens in the evening (6-7 pm). The two groups of acoustics measurements characterises the mood and movement of the sheep: in the

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morning the sheep are fresh from their rest and are walking down-hill with a slope of 10 degrees and average speed of $2.4 \pm 0.2 \text{ m.s}^{-1}$, whereas in the evening the sheep are tired and walking up-hill at an average speed of $1.2 \pm 0.2 \text{ m.s}^{-1}$. The measurement is made over seven consecutive days. It is found that speeds are in good agreement with the kinetic characteristic of the walking gate of a sheep as measured on a pressure sensing walking way [4].

When the clapper is struck against the rim, the metal-on-metal impact imparts energy instantaneously as sound travels through metal at approximately 5130 m.s⁻¹) into the bell. At this moment a temporary distortion (hum) of the rim occurs from the where the energy is transmitted throughout the bells to produce a continuous succession of partials resonate tones. It is this time dependent combination of strike tone and partials which gives rise to the timbre [5] of the bell. However and unlike tuned cast metal hand-bells and churchbells, the sheep bell's elliptical shape and composite design imparts boundaries on the transmission of energy throughout the met of the bell due to the stiffness at the two welded acute angles on the major axis and the less stiff regions on the minor axis. In addition the sheep bells perceived timbre also depends on whether the struck region of the rim is damped by the sheep's neck. Thus the mechanical interaction within a sheep bell's timbre is potentially more complex, in both pitch and amplitude, when compared to a tuned cast bell.

2. Experiment

2.1 Sheep bell

The sheep bell studied here is of the composite elliptical open-bell design that is made from two formed iron metal sheet (1 mm thick) that are brazed/welded together to form the elliptical shaped aperture behind which a closed air-column acoustic chamber is formed. The edge dimensions of the aperture are typically 11.5 cm between the welded seams and 16 cm from the rim to the bell node. The clapper is made from 4 mm diameter x 10 cm in length.



Figure 1a and b: a) photograph of flat side of sheep bell, b) Schematic view of open aperture of sheep bell.

Finally the bell's nodal point (minimum vibration point) is attached to the sheep using a leather strap. A photograph of a sheep bell is shown in Fig 1a and a schematic of the aperture of the bell are shown in Fig 1 b.

2.1 Acoustic recording

The sound recording and deconvolution analysis used in this study is performed by National instrument LabVIEW 20011 software program running on a Dell laptop. This software has been published elsewhere [6-10]. The recordings where made using an Omi-directional condenser microphone and sampled at rate of 24000 samples per second with a 24 Bit depth for a period of 1 second. In all cases the measurement where made at a distance of 20 m from the noise source (bell and sheep herd). In order to identify the timbre of the sheep bell minor and major axis, a single bell was isolated and freely suspended and the clapper struck using the force of a human hand. Here we define to frequency that go make the timbre are in the normal healthy human hearing frequency is between 20 Hz to 20 kHz, but is far more in the 1 to 4 kHz [5]. These two recordings along with a recording of the surrounding acoustic environment (baseline) are used as sound references for subsequent decoding of the sheep traveling upon the road. To standardize the reference measurements with the sheep acoustic recordings a piece-by-piece Savitzky-Golay (SG) [11] moving window of 10 Hz is used smooth the amplitude of the time series data. This digital conditioning of the recordings matches the same conditioning process to remove the high frequency sound of the sheep's feet impacting on the concrete road surface.

3. Results

Three sets of 10 bell recordings were made. These sets are reported in sections: 3.1 for a single bell removed from sheep neck and struck by a human hand, 3.2 bells attached to 3 sheep within the herd as the sheep are walking up-hill (evening), and 3.3 as the sheep herd are walking down-hill in the morning. In all three cases the recording microphone is placed 20 m perpendicular to the direction of the herd movement.

3.1. Sheep bell response (freely suspended)

Figure 2 shows a triplet of reference acoustic spectra for the freely suspended sheep bell. The top spectrum is associated with the clapper striking the bell on the major axis, the middle spectra is associated with the clapper striking on the minor axis and lower spectrum is a measurement of the surrounding area without any strikes (baseline) and is only shown for comparative purpose here.

Upon comparison of the spectra's, there a number of features of note: Firstly the strike tone is seen to formed from two peaks with frequencies of 600 Hz 740 Hz which is followed by harmonic related overtones/partials that exhibit dual picks, The overtones/partials frequencies in the major axis spectra appears to have a strong odd harmonic relationship to the strike tone. For example n = 3, 5,

7, 9 etc., whereas both even overtones/partial frequencies (n = 2, 4, 6, 8 etc.) and odd overtones appear in the minor axis spectra appears. The disparity that appearance the between odd and odd plus even overtones leads the minor axis spectra having a richer timbre which may be expressed by the normalised mean amplitude (centred around ±200 Hz) of the even overtones/partials to the strike tone amplitude as denoted using the standard notation of loss to the carrier (dBc), see the annotated dashed box for n = 2, 4, 6 and 8 in figure 2 and measured mean values in table 1. In table 1 it can be seen that the loss to the strike tone (carrier) for n = 4, 6, 8 is greater for the major axis typically 52.6 dB as compared to typically 41.6 dB for the minor axis. The lost however at n = 2 is reversed but only by 3 dB.



Figure 2: Frequency response of sheep bell as struck on the major, minor axis using a human hand and basline measurement.

Table 1. Strike tone amplitude and dBc values centred (±200 kHz) around even overtones/partials for both major and minor axis.

Axis	Strike tone	n = 2	N = 4	n = 6	n = 8
	(dB)	(dBc)	(dBc)	(dBc)	(dBc)
Major axis	-41	-44	-50	-52	-56
Minor axis	-47	-48	-43	-37	-45

To a first approximation, the strike tone frequency (f_o) and the odd overtone/partials frequencies (f_n) may be represented mathematically using a standing-wave quarter wavelength closed air-column model [5-9] as shown equation (1).

$$f_n \approx \frac{nc}{4L} \tag{1}$$

In equation (1), *n* is modulo frequency number, *L* is the physical length of resonator and *c* is the sound velocity at 20 °C (air: ~346 m.s⁻¹; iron: ~5130 m.s⁻¹). For a closed air-column, the bell aperture defines the antinode (maximum pressure vibration) and the node point defines the minimum vibration point. Thus using equation (1), the bells 600-800 Hz strike tones equates to L = 11.4 to 11.8 cm. Using the bell's geometric information proved in figure 1 the computed value of *L* approximates to the bell's major axis and either side of the apertures surface length which would suggest the twin peaks in the strike tone originate from the aperture volume and the metal rim, Given this configuration, odd overtones/partials are readily supported and even overtones/partials are suppressed. To predict both even and odd overtones/partials resonances equation 1 needs to modified by replacing the 4 the denominator with 2 thus making equation 1 to represent a half wavelength resonator.

3.2. Sheep bell response as the herd is moving up-hill

Figure 3 shows a representative acoustic frequency spectrum of the 10 recordings of 3 similar bells (with a major axis of 10 cm) attached to 3 individual sheep within the herd. The herd are being walked up-hill at an average speed of 1.2 m.s^{-1} . The acoustic spectrum shows a clear strike tone at 750 Hz followed by a series of harmonically grouped overtone/partials at 2.3 to 2.66 kHz, 4.08 to 4.85 kHz. The dispersion frequency spans of these of these groups are of the order of 1 kHz.

The frequency position of the overtones/partials reveal two features of note. Firstly it is known that sonic energy travels approximately 14 times faster through ion when compared to air, which will result partials having a different harmonic relationship to the strike tone. Secondly, the odd harmonic relationship of the reference bell, as discussed in section 3.1, is observed; in that they have an odd harmonic relationship (n = 3 (2.55 kHz); n = 5 (4.25 kHz); and n = 7 (5.95 kHz) to the strike tone. Using equation 1 the frequency of the strike

tone corresponds to the characteristic bell length of 10 cm. However, the overtones appear to have twice the bandwidth (~1 kHz) as compared to the reference iron bell (~0.5 kHz).



Figure 3: Frequency response of sheep herd walking up-hill at 1.2 m.s⁻¹.

3.3. Sheep bell response as the herd is moving down-hill

Figure 4 shows a representative frequency signature of the 10 acoustic recordings of the same herd with three bells, but as they are walking down-hill in the morning at an average speed of 2.4 m.s⁻¹. When compared to the sheep walking up-hill spectrum (figure 3) the recorded spectrum shows that the strike tone, a 425 Hz tone and overtones/partials are present but there is a significant increase in the number of discrete and irregular frequency spaced (10 to 100 Hz) noise (14 to 20 dBc) peaks between the bell's strike tone and the 3rd overtone/partial without altering the frequency dependent noise floor level at the even harmonics (n = 2 (1.7 kHz); n = 4 (3.4 kHz); and n = 6 (5.1 kHz)) locations.

Demodulation $(^{1}/\Delta_{f} = s)$ of the irregular frequency response between the strike tone and the 3rd overtone provides a characteristic time of 0.01 to 0.1 seconds. Given the factor of 2 increase in herd speed between the up-hill (figure 3) and down-hill (figure 4) recordings, the additional irregular peaks may originate from the impact of the sheep's trotters are being picked-up by the microphone.

To analysis the gait of the sheep an audio-visual movie was made for both the upward and downward directions of the herd. It was found that the sheep have a two-beat diagonal gait (trot) where the diagonal pairs of legs move forward at the same time in the down-hill case (2.4 m.s^{-1}) . In the up-hill case

 (1.2 m.s^{-1}) the sheep tend to move one leg at time. This result is in good agreement with the work of J. Kim and G. Breur who used a pressure sensing walkway to measure the gait of Suffolk-mix sheep [4]. In their work it was reported that the walking trot gait imparted 50-56% of the sheep's body weight to the synchronised diagonal forward and hind limb with a disparity of 59% to 41% in favour of the forward limb. This would imply the loudness of the sheep trot signature would be greater than the up-hill walking gait where one limb is moved at a time. It is presumed that in our case the loudness (noise) of the sheep walking up-hill gait is not observed due to the noise floor of the acoustic measurement.



Figure 4: Frequency response of sheep herd walking down-hill at 2.4 m.s⁻¹.

4. Conclusion

Acoustic recordings of iron composite bells have been made in the frequency range of 0 to 8 kHz. The acoustic signature of single (and isolated) reference bell is used to identify the bells strike tone and overtones/partials response when the clapper struck against the metal rim. It is found the bell supports odd overtones/partials and abates the even overtones/partials. These recordings are then used as a reference to decode the frequency dependent acoustic signature of bells attached to 3 sheep within a herd of 27 to 30 healthy male and female Sfakia sheep as they are walked up-hill and down-hill on an inclined (10%) concrete road at a pace of 1.2 and 2.4 m.s⁻¹, respectively.

Time series analyses of the acoustic recordings of the herd indicate that there is significant difference in the up-hill and down-hill. The difference in acoustic signature is attributed to the change in the walking gait of the sheep: from onebeat to two-beat impact as the sheep alter their gait from a walk to a trot. Acoustically the difference occurs in the 850 Hz to 2.6 kHz frequency range which is the sensitive hearing range of the human ear.

It is concluded that the movement behaviour of a sheep herd that lay between the extremes of grazing and predator threat can be discriminated using the non-obtrusive and non-worrying technique of acoustic recording.

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Spatial and temporal imaging of a plasma jet plume

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Abstract: Cold atmospheric pressure plasma jets have been shown to exhibit considerable potential for use in plasma medicine applications such as in wound treatment. New pulsed atmospheric pressure plasma jets are being developed that have inherent plasma stability and low gas temperatures. In this study examines a new digital enhancement technique to characterise the far field plasma plume and effluent region of the plasma. The digital technique provides spatial information that identifies possible gas treatment zones for medical applications. Using images from a fast a capture (10 μ m second) ICCD camera the study shows the luminous plume extends up to 7 mm from the reactor exit nozzle and has a kinked, or wrinkled, appearance but nonluminous perturbation of the gas is detected up to 3 cm away to the front and either side of the visible plasma plume.

Keywords: Imaging, Atmospheric plasma jet; Diagnostics.

1. Introduction

The development of the cold temperature atmospheric pressure plasma jets in recent years has led to the promising new science of plasma medicine. Treatments are generally applied using a hand-held atmospheric plasma sources that utilise a wide range of electric drive frequencies and reactor geometries. Examples of cell treatment leading to apoptosis using these plasma jets have been reported by a number of authors [1, 2]. One of the first clinically proven hand-held plasma jets is the kINPen med® developed by the Leibniz Institute for Plasma Science and Technology (INP), Greifswald, Germany in cooperation with neoplas GmbH, Greifswald, Germany is now undergoing in-vivo clinical investigation of plasma antiseptic properties on human skin [3], chronic venous leg ulcers [4] and cosmetic surgery [5]. These clinical trials require the relatively small 1.6 mm diameter plasma to treat large areas of thermally sensitive living tissue and microorganisms. Earlier studies using the kINPen 0.9 versions [6-8] of the plasma jet on microorganism have shown that cells are killed outside the visible plasma plume immediate treatment area, indicating what has been termed a 'spillover' occurs [9]. Further to this, kINPen med® plasma induced activation studies on poly(ethylene-terephthalate) PET at a nozzle-to-surface distance of 5-15 mm have shown that a similar immediate activation (1 day) post treatment 'spillover' can be induced up to 20 mm in

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diameter on the polymer surface [10].

This work reports on the spatial and temporal visual imaging of the kINPen med® plasma plume fluid structure using a photodiode (PD) to trigger a gated ICCD camera, with the addition of a new digital image processing technique of the ICCD camera images. This post image processing technique is used to enhance the immediate area (up to a distance of approximately 3 cm) around the luminous plasma plume to reveal the fluid structure emanating from the gas flow. This digital image enhancement approach differs from the Schlieren imaging technique [11, 12] (where an image is obtained by illuminating a test volume with parallel rays (collimated from a point source) that are then brought through a focus at which a knife edge cuts off half the field and most of the focal spot) in that all the image processing is performed on the fast ICCD camera only. In this work the widely available National Instrument LabVIEW software packages is used as an example. The plasma imaging also differs from the high temporal resolution flame-front visualization technique [13] were both broadband and narrowband spectra images have been reported without colour plane extraction. This work also differs from large time scale (10s) imaging of the influence of a rotating electrode within DBD plasma has been reported and shown to generate complex vortex flow structures at the dielectric surface [14].

2. Experiment apparatus and methods

Figure 1a shows a photograph of the plasma jet used in this study. The plasma reactor is a cylindrical dielectric barrier discharge made from a glass ceramic with an internal diameter of D = 1.6 mm. The inner metal electrode has a diameter of ~0.3 mm. The outer body is grounded to produce a cross-field jet configuration i.e. an electric field perpendicular to the gas flow. Here a gas flow rate of 5 SLM of 99.99% pure argon is used, equating to a gas velocity through the reactor tube of v = 36.7 m.s⁻¹. Since the plasma region is 20 mm long there is a gas residence time of about 0.5 ms.



Figure 1: Photograph of the kINPen Med® plasma interacting with a fingertip.

The inner electrode is powered by a 1 MHz electrical drive frequency that is pulse modulated with 2.5 kHz square wave (50 % duty cycle) signal [10]. In this

work the plasma plume, and effluent expands unobstructed into atmospheric pressure air, and is investigated with a PD, fast imaging camera and post image capture enhancement technique to reveal the fluid structure around the plasma plume (0-30 mm) in front and to the side of the plume.

The PD used was a Hamamatsu MPPC with a rise time of 10 ns and spectral range between 320 and 900 nm [15]. The light was collected at right angles to the plume, 1 mm downstream of the nozzle exit, via a fibre optic with a collimating lens, the combination producing a focal area of 1 mm in diameter at a length of 6 mm from the lens: Thus making the interrogation area smaller than to the diameter of the jet discharge (~1.6 mm). The rising edge of the 2.5 kHz modulated plasma light is used to trigger the ICCD Camera.

The Andor iStar 334T ICCD camera is used to capture the plasma images. A 14 cm focal length glass lens focused the region from between 2 mm upstream to 20 mm downstream of the exit nozzle. Using this combination the overall optical chain (between camera and plasma-plume) is of the order of 2 m and the camera spectral range is restricted to 300 to 850 nm by the glass lens. The camera was triggered, via a delay generator, from the rising edge of the PD signal. Within the camera the images are processed using a false-colour scale from blue (low intensity) to yellow (high intensity) for maximum visual differentiation the gain was set to 2817 out of a maximum of 4095, where the final digital images are formatted as a 24-bit red-green-blue (RGB) JPEG (Joint Photographic Experts Group) with a N x N pixel array, where N = 1024. Through an initial survey of the pulse-on and pulse-off periods of plasma the ICCD was synchronised to the respective time periods.

The gas behaviour beyond the luminous plasma region is explored by using LabVIEW based software [16]. This software essentially extracts the lowest intensity colour plane (blue plane) from the original RGB image and then uses pixel resolution enhancement through digital filtering and a thresholding algorithm. Care is taken at each step to ensure that the morphology in the recorded data is not distorted by reference at each step to experimentally available information, and the goals of the operation and limitations of the algorithms. The final images were achieved using four standard sequential steps.

- 1. The 8-bit "blue" plane is selected from the original 24-bit RGB image.
- 2. A fast Fourier Transform (FFT) is then applied to this plane to convert the spatial information into its frequency domain. A low-pass filter is used to smooth the noise with a truncation process to remove any remaining high frequency component above the user defined cut-off point.
- 3. An inverse FFT is then applied to bring the frequency domain data back into the spatial domain.
- 4. A local Nibalck thresholding segmentation algorithm is then used to produce a binary image. In this operation the background particles are set to I = 0 (black) while setting fluid structure to a pixel value of I = 1 (white). The result of this process produces a black-and-white binary image that represents the fluid structure within the original blue image.

3. Results.

3.1 Visible plasma imaging

Figures 2 provide examples of 15 individual ICCD images sampled from 31 images obtained for the argon plasma. The images span from the beginning of the pulse at $t = 0 \ \mu s$ to the end stages of the pulse at, $t = 185 \ \mu s$. With the gain fixed at 2817 each image has the same intensity scale and therefore their intensities may be compared directly. To add comparison a scale bar is displayed at the top of the figure. The figure shows a linear increase in the length of the plume between 0 μs to 40 μs and rapid decrease in length beyond 185 μs when the plume is almost completely gone. Apart from the earliest and latest times the plumes vary in visible length and exhibit a kinked or wrinkled structure along the length of each plume.



Figure 2: A selection (a total 15) of space and time resolved images of the nozzle and argon plasma.

Using all the 31 ICCD images, the distal length of each discharge plume have been calculated but are not shown here. The calculations reveal that the plasma expands from the nozzle and reaches, and maintains, a maximum length of about 4.5 or 6 mm until the voltage pulse is terminated. The initial velocity of the visible plume front is about 200 m.s⁻¹. However at about 4.25 mm the argon the front rapidly accelerates to about 300 m.s⁻¹ before reaching its maximum length with a periodic cycling ranging from 6.5 to 5 mm: with each cycle period taking 40 to 45 microseconds, which equates to a frequency of 20 to 22 kHz.
3.2 Spatial enhancement of non-visible region around plasma plume

Figure 3 show a screen shot image of the LabVIEW colour plane extraction and line profile front panel for the pulse-on period. In this figure the left-hand images is the original 32-bit image with interactive line intensity profile (LIP) cursor; the second column of images are the three extracted blue, green and red planes (presented here in grayscale); the third column of graphs depict the selected LIP for each plane; and the final column is basic descriptive analysis of the LIP for each plane. The information presented on this front panel reveals that majority of the plasma information (white to grey colours) is aligned along the flow of the plume in the red and green planes. In contrast the far-field low intensity fluid structure information is captured within the blue plane as speckled noise surrounding the plume with an outer white ring at a typical distance of 2-4 plume diameters either side of the plume.



Figure 3: LabVIEW RGB colour plane and line profile.

We now turn to the digital filtering and threshold processing of the blue image. Figure 4 shows the processing of the duration of the pulse-on period and the duration of pulse-off period. It is interesting the structure observed on short time scale (figures 2) is absent in the long exposure image. It is also apparent from figure 4b that there is afterglow. In figure 4c we are imaging the structure of the background gas. This shows a distinct ripple-like feature centred in the proximity of the maximum light emission from the plume. Figure 4f show that this is absent when there is no discharge present.

To understand these fluid structure images we consider the dimensionless Reynolds number (R_e) as defined in equation 1 when interpreting figures 4c and

4f, as it provides a measure of the ratio of inertial forces to viscous forces and quantifies the relative importance of these two types of forces.

$$R_{e} = \frac{QD}{v_{k}A} \tag{1}$$

Where Q is the neutral argon flow rate (8.35 x 10-5 m³.s⁻¹), D is the diameter of the nozzle (0.0016 m), v_k is the gas kinematic viscosity (0.000014 m².s⁻¹) and A is the cross sectional area of the nozzle (2 x 10⁻⁶ m²). For argon gas flows of 5 SLM, R_e equates to 4465 which implies the inertial forces are expected to be more dominant than viscous forces and large-scale fluid motion would be undamped in the pulse-off period. However when the plasma is turn-on, the neutral gas flow rate will increase due to the associated gas heating.



Figure 4: 0.2μ second exposure ICCD images of plasma in pulse-on (a) and pulse-off period (d); images (b) and (c) depict the image enhancement of the pulse-on period; and images (e) and (f) depict the image enhancement for the pulse-off period.

Considering the processed image of the pulse-on period (figure 4c) a ripple structure is observed to radiate from a point along the axis of the plasma plume and extends with a complex structure in the direction of effluent flow up to 4 cm from the nozzle. This repeating far-field wave-like structure with a white peak distances separation of typically 1-2 mm is within an order of magnitude of the expected travel distance of the neutral gas within the capture time-frame of the camera image. In addition the ripple pattern is found to be asymmetric with respect to the effluent flow axis, producing a complex broken structures to the top beyond which the discontinuities the structures extend into the ambient air. The distance occurs at around 0.5 cm from the plume distal point.

In the case of the pulse-off period (figure 4f) the wave-like structures has collapsed to form irregular and small-scale chaotic structures with scale lengths of the order of the nozzle diameter. These observations are consistent with the Reynolds number dimensionless analysis and the loss of driving force to heat the plasma gas when the electrical drive power is switched-off. Under these conditions the heated gas is expected to begin to equilibrate with the surrounding ambient air.

4. Conclusion

The spatial and temporal visual imaging of an argon-based pulsed plasma jet designed for medical use has been studied using photodiode and ICCD camera imaging, plus post exposure enhancement of the camera images. This combined measurement and diagnostic approach provides a spatial and temporal picture of the plasma plume and its effluent. The PD measurements show that the plasma is modulated by a fast rising and falling 2.5 kHz square wave time-base profile. Microsecond time scale imaging of the discharge using the ICCD camera reveals that the plasma plumes are continuous through the 0.2 ms pulse-on period of the discharge. However the plume morphology takes on a kinked or wrinkled appearance. In addition the plume rapidly decays at the end of the voltage pulse suggesting micro-turbulence is the driving force in the production of the kinks within the plasma jet To gain access to the effluent gas being expelled from the plasma plume the technique of image plane extraction has been developed and demonstrated. Here the blue plane of the ICCD digital images has revealed pulsed plasma induced fluid structures extending up to 2-3 cm form the visible plume. This far-field fluid structure information may be used in the understanding 'spillover' effect when plasma treating thermally sensitive polymers and their biomaterial counter parts.

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The Resonances and Poles in Isoscattering Microwave Networks and Graphs

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Abstract. "Can one hear the shape of a graph?" - this is a modification of the famous question of Mark Kac "Can one hear the shape of a drum?" which can be asked in the case of scattering systems such as microwave networks and quantum graphs. It addresses an important mathematical problem whether scattering properties of such systems are uniquely connected to their shapes? Recent experimental results of Hul et al. [1], Lawniczak et al. [2] and Lawniczak et al. [3] based on a characteristics of graphs such as the cumulative phase of the determinant of the scattering matrices indicate a negative answer to this question. In this presentation we review new important results devoted to the isoscattering networks which are based on local characteristics of graphs such as structures of resonances and poles of the determinant of the scattering matrices [3]. Using the analytical formulas for the elements of the scattering poles of the determinant of the scattering matrices we show that it is possible to link the structure of the scattering poles of the microwave networks. Furthermore, we show that theoretically reconstructed spectra of the networks are in good agreement with the experimental ones.

Keywords: Quantum and classical chaos, Isoscattering systems, Microwave networks and quantum graphs, Microwave and quantum billiards, Open systems.

1 Introduction

The famous question posed by Marc Kac in 1966 "Can one hear the shape of a drum?" [4] addresses the problem whether two isospectral drums have the same shape. In general, two vibrating systems are isospectral if and only if their spectra are identical. In mathematical terms Marc Kac's question reduces to a question of uniqueness of spectra of the Laplace operator on the planar domain with Dirichlet boundary conditions. The negative answer to the above question was given in 1992 by Gordon, Webb, and Wolpert [5,6]. Using Sunada's theorem [7] they found a way to construct pairs different in shape but isospectral domains in \mathbb{R}^2 . The procedure of designing isospectral planar domains consists of cutting the 'drum' into subdomains and rearranging them into a new one with the same spectrum. An experimental confirmation that

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'hearing' the shape is impossible was presented by Sridhar and Kudrolli [8] and Dhar et al. [9].

The problem of isospectrality for quantum graphs was considered by Gutkin and Smilansky [10]. Quantum graphs consist of one-dimensional bonds which are connected by vertices. The wave propagation in each bond is governed by the one-dimensional Schrödinger equation. Gutkin and Smilansky proved that the spectrum identifies uniquely the graph if the lengths of its bonds are incommensurate. A general method of construction of isospectral graphs [11,12] uses the extended Sunada's approach. In this method one cuts the graph and "transplants" the pieces into a different arrangement. As a result of the transplantation every eigenfunction of the first graph one can assign an eigenfunction of the second one with the same eigenvalue.

However, inability of determining the shape from the spectrum alone does not preclude possibilities of distinguishing one drum from another in scattering experiments. Basing on numerical simulations Okada et al. [13] showed that isospectral domains constructed by Gordon, Webb and Wolpert can be distinguished in scattering experiments by different distributions of poles of the scattering matrices. Therefore, one can pose an important question whether also the geometry of a graph can be determined in scattering experiments.

This question was answered negatively by Band, Sawicki and Smilansky [14,15]. They analyzed isospectral quantum graphs with attached infinite leads which are called isoscattering. In [14,15] the authors showed that any pair of isospectral quantum graphs obtained by the method outlined in [11,12] is isoscattering if the infinite leads are attached in a way preserving the symmetry of the isospectral construction [14,15].

By definition, isoscattering graphs are isopolar when their scattering matrices have the same poles or isophasal when the phases of the determinants of their scattering matrices are equal.

Isopolar lossless graphs need not be isophasal since to determine the phases one needs more information. In contrary, any two isophasal lossless graphs are isopolar [3].

2 Quantum graphs and microwave networks

Quantum graphs can be treated as idealizations of physical networks in the limit where the lengths of the wires are much larger than their diameter. A detailed theoretical analysis of their properties as well as applications in modeling various physical problems can be found in [16] and references cited therein. Methods of their experimental realizations were presented in [17,18].

It is crucial for this work that quantum graphs can be successfully modeled by microwave networks [19]. The introduction of one-dimensional microwave networks simulating quantum graphs extended substantially the number of systems which can be used to verify wave effects predicted on the basis of quantum physics. Among them the most important are highly excited hydrogen atoms [20–24] and two-dimensional microwave billiards [25–36]. The later papers on microwave networks [37–39] clearly demonstrated that they can be successfully used to investigate properties of quantum graphs also with highly complicated topology and absorption.

A microwave network consists of n vertices connected by B bonds, e.g., coaxial cables. The topology of a network is defined by the $n \times n$ connectivity matrix C_{ij} which takes the value 1 if the vertices i and j are connected and 0 otherwise. Each vertex i of a network is connected to the other vertices by v_i bonds, v_i is called the valency of the vertex i.

In the construction of microwave networks we used coaxial cables consisting of an inner conductor of radius r_1 surrounded by a concentric conductor of inner radius r_2 . The space between the inner and the outer conductors is filled with a homogeneous material having the dielectric constant ε . Below the onset of the next TE₁₁ mode [40], inside a coaxial cable can propagate only the fundamental TEM mode, in the literature called a Lecher wave.

Using the continuity equation for the charge and the current one can find the propagation of a Lecher wave inside the coaxial cable joining the i-th and the j-th vertex of the microwave network [41,19]. For an ideal lossless coaxial cable the procedure leads to the telegraph equation on the microwave network

$$\frac{d^2}{dx^2}U_{ij}(x) + \frac{\omega^2\varepsilon}{c^2}U_{ij}(x) = 0,$$
(1)

where $U_{ij}(x,t)$ is the potential difference between the conductors, $\omega = 2\pi\nu$ is the angular frequency and ν is the microwave frequency, c stands here for the speed of light in a vacuum, and ε is the dielectric constant.

If we take into account the correspondence: $\Psi_{ij}(x) \Leftrightarrow U_{ij}(x)$ and $k^2 \Leftrightarrow \frac{\omega^2 \varepsilon}{c^2}$ the equation (1) is formally equivalent to the one-dimensional Schrödinger equation (with $\hbar = 2m = 1$) on the graph possessing time reversal symmetry [42]

$$\frac{d^2}{dx^2}\Psi_{ij}(x) + k^2\Psi_{ij}(x) = 0.$$
 (2)

3 Experimental setup

Fig. 1a and Fig. 1b show the two isoscattering graphs which are obtained from the two isospectral ones by attaching two infinite leads L_1^{∞} and L_2^{∞} . Using microwave coaxial cables we constructed the two microwave isoscattering networks shown in Fig. 1c and Fig. 1d. In order to preserve the same approximate size of the graphs in Fig. 1a and Fig. 1b and the networks in Fig. 1c and Fig. 1d, respectively, the lengths of the graphs were rescalled down to the physical lengths of the networks, which differ from the optical ones by the factor $\sqrt{\varepsilon}$, where $\varepsilon \simeq 2.08$ is the dielectric constant of a homogeneous material used in the coaxial cables.

For the discussed networks and graphs we will consider two most typical physical vertex boundary conditions, the Neumann and Dirichlet ones. The first one imposes the continuity of waves propagating in bonds meeting at i and vanishing of the sum of their derivatives calculated at the vertex i. The latter demands vanishing of the waves at the vertex.

The graph in Fig. 1a consists of n = 6 vertices connected by B = 5 bonds. The valency of the vertices 1 and 2 including leads is $v_{1,2} = 4$ while for the other ones $v_i = 1$. The vertices with numbers 1, 2, 3 and 5, satisfy the Neumann vertex conditions, while for the vertices 4 and 6 we have the Dirichlet ones. The graph in Fig. 1b consists of n = 4 vertices connected by B = 4 bonds. The vertices with the numbers 1, 2 and 3 satisfy the Neumann vertex conditions while for the vertex 4, the Dirichlet condition is imposed.

The bonds of the microwave networks shown in Fig. 1c and Fig. 1d have the following optical lengths:

$$a = 0.0985 \pm 0.0005$$
 m, $b = 0.1847 \pm 0.0005$ m, $c = 0.2420 \pm 0.0005$ m, $2a = 0.1970 \pm 0.0005$ m, $2b = 0.3694 \pm 0.0005$ m, $2c = 0.4840 \pm 0.0005$ m.

In order to properly describe considered by us systems we use the two-port (2×2) scattering matrix

$$S(\nu) = \begin{pmatrix} S_{1,1}(\nu) & S_{1,2}(\nu) \\ S_{2,1}(\nu) & S_{2,2}(\nu) \end{pmatrix},$$
(3)

relating the amplitudes of the incoming and outgoing waves of frequency ν in both leads.

The two-port scattering matrix $S(\nu)$ was measured by the vector network analyzer (VNA) Agilent E8364B. The VNA was connected to the vertices 1 and 2 of the microwave networks which are shown in Fig. 1c and Fig. 1d. The scattering matrix $S(\nu)$ was measured in the frequency range $\nu = 0.01 - 2$ GHz. It is important to note that the connection of the VNA to a microwave network (see Fig. 1e) is equivalent to attaching of two infinite leads to a quantum graph.

4 Isopolar networks

Let us remind that the two networks in Fig. 1c and Fig. 1d are isopolar if their scattering matrices have the same poles. In order to study isopolar properties of graphs presented in Fig. 1 we have to consider important local characteristics of graphs such as structures of experimentally measured resonances and theoretically evaluated poles of the determinant of the two-port scattering matrices. Such an analysis is important since for open systems resonances show up as poles [43,44] occurring at complex wave numbers $k_l = \frac{2\pi}{c} (\nu_l - i\Delta\nu_l)$, where ν_l and $2\Delta\nu_l$ are associated with the positions and the widths of resonances, respectively. In Fig. 2a we show that for the frequency range from 0.01 to 2 GHz the amplitudes $|\det(S^{(I)}(\nu))|$ and $|\det(S^{(II)}(\nu))|$ of the determinants of the scattering matrices $S^{(I)}(\nu)$ and $S^{(II)}(\nu)$ of the networks shown in Fig. 1c and Fig. 1d, respectively, are very close to each other, clearly showing that we are dealing with the isoscattering networks. The results obtained for the networks presented in Fig. 1c and in Fig. 1d are marked by blue full squares and red open circles, respectively.

The analytical formulas for the elements of the scattering matrices $S^{(I)}(k)$ and $S^{(II)}(k)$ are presented in the Appendix. The calculations showed that both scattering matrices posses the isoscattering properties. In Fig. 2b using the contour plot we present the poles of the amplitude of the determinant of the scattering matrix $|\det(S^{(II)}(k))|$ (solid circles) calculated for the graph with n = 4 vertices (Fig. 1b) for the frequency range from 0.01 to 2 GHz. The numerical calculations were performed for the isoscattering graph having the same bond lengths as the ones measured for the microwave network presented in Fig. 1d. We also imposed the proper vertex boundary conditions. The vertical axis of Fig. 2b shows the imaginary part $\Delta \nu$ of the poles of the graph. Fig. 2a and Fig. 2b clearly show very good agreement between the positions of the experimental scattering resonances and the theoretical poles. To make this comparison more straightforward the poles of the determinant of the scattering matrix det $(S^{(II)}(k))$ are marked in Fig. 2a by solid circles.

In general, the microwave networks are lossy. The paper [19] shows that loses in such networks can be described by treating the wave number k as a complex quantity with absorption-dependent imaginary part $\text{Im}\left[k\right] = \beta \sqrt{2\pi\nu/c}$ and the real part $\text{Re}\left[k\right] = 2\pi\nu/c$, where β is the absorption coefficient and c is the speed of light in vacuum. The analytical formulas for the theoretical scattering matrices $S^{(I)}(k)$ and $S^{(II)}(k)$ allow us to reconstruct the resonances in the amplitudes of the determinants of the scattering matrices. Since the graphs are isoscattering both theoretical reconstructions give exactly the same results. The solid line in Fig. 2a shows the amplitude of the determinant of the scattering matrix $|\det(S^{(II)}(k))|$ calculated for the absorption coefficient $\beta = 0.00762m^{-1/2}$. Fig. 2a shows that the theoretical results are in very good agreement with the experimental ones.

In summary, we analyzed resonances of the two microwave networks which were constructed to be isoscattering [3]. We showed that the networks are iospolar, i.e., isoscattering, within the experimental errors. Therefore, the question "Can one hear the shape of a graph?" is answered in negative.

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5 Appendix

For brevity of notation we denote the wave propagating through an edge (bond) e by Ψ_e , i.e. use edges to index the waves rather than corresponding vertices as in Eq. (2). Propagation in each edge is described by the free Schrödinger equation

$$-\frac{d^2}{dx_e^2}\Psi_e(x_e) = k^2\Psi_e(x_e),\tag{4}$$

where x_e is a coordinate parameterizing the edge e. The propagation in the whole graph is governed by the Laplace operator on the graph which is the sum of one-dimensional Laplacians, $-d^2/dx_e^2$, each acting on the corresponding edge.

The solution of (4) for each edge takes the form

$$\Psi_e(x_e) = a_e^{in} \exp(-ikx_e) + a_e^{out} \exp(ikx_e).$$
(5)

Also for the two leads L_1^{∞} and L_2^{∞} , we have

$$\Psi_l(x_l) = a_l^{in} \exp(-ikx_l) + a_l^{out} \exp(ikx_l), \quad l = 1, 2.$$
(6)

If the wave with a wave number k propagates in the whole graph, i.e. k^2 is an eigenvalue of the graph Laplacian (for a scattering graph the spectrum of eigenvalues is, in general, continuous), the solutions (5) and (6) satisfy the vertex boundary conditions for a particular graph. Imposing the conditions, we obtain a linear set of equations connecting the amplitudes a_e^{in} and a_e^{out} of the forward and backward propagating waves in the edges, as well as incoming and outgoing amplitudes a_l^{in} and a_l^{out} in the leads. The equations can be solved for $a_{1,2}^{out}$ in terms of $a_{1,2}^{in}$,

$$\begin{pmatrix} a_1^{out} \\ a_2^{out} \end{pmatrix} = \begin{pmatrix} S_{1,1}(k) \ S_{1,2}(k) \\ S_{2,1}(k) \ S_{2,2}(k) \end{pmatrix} \begin{pmatrix} a_1^{in} \\ a_2^{in} \end{pmatrix}.$$
 (7)

Applying this procedure to the graphs in Fig. 1a and Fig. 1b we get, respectively

$$\begin{split} S_{1,1}^{(I)}(k) = \\ & \frac{\left(-1+e^{4ibk}\right)\left(-1+e^{4ick}\right)-2\left(-1+e^{4i(b+c)k}\right)\cos\left(2ak\right)-2i\left(-1+e^{2i(b+c)k}\right)^{2}\sin\left(2ak\right)}{\left(-3+e^{4ibk}+e^{4ick}+e^{4i(b+c)k}\right)\cos\left(2ak\right)-i\left(-5+e^{4ibk}+e^{4ick}+4e^{2i(b+c)k}-e^{4i(b+c)k}\right)\sin\left(2ak\right)}, \\ & S_{1,2}^{(I)}(k) = \\ & \frac{2\left(-e^{2ibk}-e^{2ick}+e^{2i(2b+c)k}+e^{2i(b+2c)k}\right)\sin\left(2ak\right)}{i\left(-3+e^{4ibk}+e^{4ick}+e^{4i(b+c)k}\right)\cos\left(2ak\right)+\left(-5+e^{4ibk}+e^{4ick}+4e^{2i(b+c)k}-e^{4i(b+c)k}\right)\sin\left(2ak\right)}, \\ & S_{2,1}^{(I)}(k) = \\ & \frac{-\left(-1+e^{4ibk}\right)\left(-1+e^{4ick}\right)-2\left(-1+e^{4i(b+c)k}\right)\cos\left(2ak\right)-2i\left(-1+e^{2i(b+c)k}\right)^{2}\sin\left(2ak\right)}{\left(-3+e^{4ibk}+e^{4ick}+e^{4i(b+c)k}\right)\cos\left(2ak\right)-i\left(-5+e^{4ibk}+e^{4ick}+4e^{2i(b+c)k}\right)e^{3in\left(2ak\right)}}, \end{split}$$

for the graph 1a, and

$$\begin{split} S_{1,1}^{(II)}(k) = \\ - \frac{2i\sin\left((b+c)k\right)\left[\left(1-e^{4iak}\right)\cos\left((b-c)k\right) + \left(1+e^{4iak}\right)\cos\left((b+c)k\right) - i\left(1-e^{4iak}\right)\sin\left((b+c)k\right)\right]}{1-e^{4iak} + \cos\left(2(b-c)k\right) + \left(-2+e^{4iak}\right)\cos\left(2(b+c)k\right) + 2i\sin\left(2(b+c)k\right)}, \\ S_{1,2}^{(II)}(k) = \frac{2e^{2iak}\sin(2bk)\sin(2ck)}{1-e^{4iak} + \cos(2(b-c)k) + \left(-2+e^{4iak}\right)\cos\left(2(b+c)k\right) + 2i\sin(2(b+c)k)}, \\ S_{2,1}^{(II)}(k) = S_{1,2}^{(II)}(k), \\ S_{2,2}^{(II)}(k) = \\ - \frac{2i\sin\left((b+c)k\right)\left[\left(-1+e^{4iak}\right)\cos\left((b-c)k\right) + \left(1+e^{4iak}\right)\cos\left((b+c)k\right) - i\left(1-e^{4iak}\right)\sin\left((b+c)k\right)\right]}{1-e^{4iak} + \cos\left(2(b-c)k\right) + \left(-2+e^{4iak}\right)\cos\left((b+c)k\right) - i\left(1-e^{4iak}\right)\sin\left((b+c)k\right)\right]} \end{split}$$

for the graph 1b.

The optical lengths of the bonds of the microwave networks are denoted respectively by a, b, and c.

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Fig. 1. A pair of isoscattering quantum graphs and the pictures of two isoscattering microwave networks are shown in the panels (**a-b**) and (**c-d**), respectively. Using the two isospectral graphs, (**a**) with n = 6 vertices and (**b**) with n = 4 vertices, isoscattering quantum graphs are formed by attaching the two infinite leads L_1^{∞} and L_2^{∞} (dashed lines). The vertices with Neumann boundary conditions are denoted by full circles while the vertices with Dirichlet boundary conditions by the open ones. The two isoscattering microwave networks with n = 6 and n = 4 vertices which simulate quantum graphs (**a**) and (**b**), respectively, are shown in the panels (**c-d**). The connection of the microwave networks to the Vector Network Analyzer (VNA) was realized by means of the two microwave coaxial cables (see panel **e**).



Fig. 2. (a) The amplitude of the determinant of the scattering matrix obtained for the microwave networks with n = 6 (blue full squares) and n = 4 (red open circles) vertices. The solid line shows the resonances of the amplitude of the determinant of the theoretically evaluated scattering matrix for the quantum graph with n = 4vertices. The results are presented in the frequency range 0.01-2 GHz. The positions of the theoretical poles (see panel (b)) are marked by big solid circles. The right vertical axis of Fig. 2a shows the imaginary part $\Delta\nu$ of the poles of the graph. (b) The contour plot shows the positions of scattering poles of the amplitude of the determinant of the theoretically evaluated scattering matrix for the quantum graph with n = 4 vertices.

Comparison of non-relativistic and relativistic Lyapunov exponents for a low-speed system

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Abstract. The Newtonian and special-relativistic Lyapunov exponents are compared for a low speed system – the periodically-delta-kicked particle. We show that although the agreement between the Newtonian and special-relativistic transient Lyapunov exponents rapidly breaks down initially, they converge to values which are very close to each other. **Keywords:** kicked particle, Lyapunov exponent, special relativity, Newtonian approximation

1 Introduction

It is conventionally believed [1-3] that if the speed v of a dynamical system is *low* compared to the speed of light c, that is, $v \ll c$, then the specialrelativistic dynamical predictions for the system will be well-approximated by the Newtonian predictions. However, it was shown in recent numerical studies [4-9] that, contrary to the conventional belief, the agreement between the Newtonian and special-relativistic dynamical predictions for a single trajectory [4-7] and for an ensemble of trajectories [8,9] can break down completely although the speed of the system is low. Here, we extend the previous studies [4-9] to a comparison of the Newtonian and special-relativistic predictions for the Lyapunov exponent of a prototypical chaotic Hamiltonian system – the periodically-delta-kicked particle – at low speed. Details of the system and calculations will be given next, followed by the results and discussion.

2 Method

In the Newtonian framework, the equations of motion for the periodicallydelta-kicked particle are reducible to an exact mapping, which is called the standard map [10,11]:

$$P_{n} = P_{n-1} - \frac{K}{2\pi} \sin(2\pi X_{n-1})$$
(1)

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$$X_{n} = (X_{n-1} + P_{n}) \mod 1$$
(2)

where X_n and P_n are, respectively, the dimensionless scaled position and momentum of the particle just before the *n*th kick (n = 1, 2, ...), and *K* is a dimensionless positive parameter.

In the special-relativistic framework, the equations of motion for the periodically-delta-kicked particle are also reducible to a mapping, which is called the relativistic standard map [12,13]:

$$P_{n} = P_{n-1} - \frac{K}{2\pi} \sin(2\pi X_{n-1})$$
(3)

$$X_{n} = \left(X_{n-1} + \frac{P_{n}}{\sqrt{1 + \beta^{2} P_{n}^{2}}}\right) \mod 1$$
(4)

where β , like *K*, is also a dimensionless positive parameter.

The transient Lyapunov exponent for a map is generally defined [14] as

$$\lambda_n = \frac{1}{n} \ln[\operatorname{abs}(\operatorname{trace} M_n)]$$
(5)

where $M_n = J_n J_{n-1} \dots J_2 J_1$ and J_n is the Jacobi matrix. In the limit $n \to \infty$, λ_n yields [14] the largest Lyapunov exponent. A hallmark of chaos is the existence of a positive Lyapunov exponent. For the standard map in Eqs. (1) and (2), the Jacobi matrix is

$$J_n = \begin{bmatrix} 1 & -K\cos(2\pi X_n) \\ 1 & 1-K\cos(2\pi X_n) \end{bmatrix}.$$
 (6)

For the relativistic standard map in Eqs. (3) and (4), the Jacobi matrix is

$$J_{n} = \begin{bmatrix} 1 & -K\cos(2\pi X_{n}) \\ (1+\beta^{2}P_{n+1}^{2})^{-3/2} & 1-(1+\beta^{2}P_{n+1}^{2})^{-3/2} [K\cos(2\pi X_{n})] \end{bmatrix}.$$
 (7)

In each theory, the transient Lyapunov exponent [Eq. (5)] is calculated twice to determine its accuracy. The calculation for the transient Lyapunov exponent is first performed in 32-significant-figure precision and then repeated in quadruple (35 significant figures) precision. The accuracy of the transient Lyapunov exponent is determined by the common digits of the 32-significant-figure-precision and quadruple-precision calculations. For example, if the former calculation yields 1.234... and the latter calculation yields 1.235..., the transient Lyapunov exponent is accurate to 1.23.

3 Results and discussion

Here we will present an example to illustrate the typical result. In this example, $X_0 = 0.5$, $P_0 = 99.9$, K = 7.0 and $\beta = 10^{-7}$. For these initial conditions and parameters, both the Newtonian and special-relativistic trajectories are

chaotic. In this case, the speed of the particle is low, about $10^{-5}c$, up to 8800 kicks.

Fig. 1, which plots the Newtonian and special-relativistic transient



Fig. 1. Newtonian (squares) and special-relativistic (diamonds) transient Lyapunov exponents versus kick.

Lyapunov exponents for the first 30 kicks, shows that the two transient Lyapunov exponents agree with each other for the first 10 kicks but the agreement breaks down from kick 11 onwards. The agreement between the Newtonian and special-relativistic transient Lyapunov exponents breaks down rapidly because the difference between the two grows, on average, exponentially – see Fig. 2. The exponential growth constant of the difference



Fig. 2. Difference between the Newtonian and special-relativistic transient Lyapunov exponents versus kick.

between the two transient Lyapunov exponents, measured from kick 1 to kick 10, is 0.96.

However, asymptotically, the Newtonian and special-relativistic transient Lyapunov exponents converge to values which are very close to one another. In particular, at kick 8800, the Newtonian and special-relativistic transient Lyapunov exponents are both accurate to 1.27, which is quite close to the analytical estimate [10] of the asymptotic Newtonian Lyapunov exponent given by $\ln(K/2) = 1.253$. This result is surprising since the chaotic trajectories predicted by the two theories agree only for the first 16 kicks, which suggests that the two asymptotic Lyapunov exponents should not agree.

Conclusions

We have shown that although the agreement between the Newtonian and special-relativistic transient Lyapunov exponents rapidly breaks down initially, the asymptotic special-relativistic Lyapunov exponent is well-approximated by the asymptotic Newtonian value. The same result should hold for other lowspeed chaotic Hamiltonian systems since the periodically-delta-kicked particle is a prototype.

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